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Elastic instabilities and shear waves in hyperelastic composites with various periodic fiber arrangements

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We investigate the influence of fiber arrangement on elastic instabilities and shear wave propagation in hyperelastic 3D fiber composites (FCs) with periodic rectangular arrays of cylindrical fibers. We show that elastic instabilities and shear waves propagating along the fibers in uniaxially deformed FCs can be tuned through the choice of the periodicity of fiber arrangements (or periodic unit cell aspect ratio b/a) of FCs. In particular, we find that the range of deformations, where FCs are mechanically stable, decreases with an increase in the periodicity aspect ratio. Moreover, we identify the bounds for the critical stretches in FCs with periodic rectangular arrays of fibers. We find that the FC critical stretches are bounded by the critical values for corresponding 3D laminates (upper bound) and FCs with square arrays of fibers (lower bound). In FCs with large volume fractions of fibers (e.g. 25%), the polarization directions of the long shear waves start rotating upon approaching critical deformation. This indicates that fibers develop buckling shapes in non-principal planes. In FCs with small volume fractions of fibers (e.g. 5%), polarizations of the shear waves (of the lowest frequencies) barely changes the direction upon attaining the critical deformation; hence, buckling of fibers initially develops in one of the principal planes, depending on the periodicity aspect ratio of FCs.

1. Introduction

Soft fiber-reinforced composites (FCs), simultaneously possessing high strength, lightweight, and flexibility, are widely present in nature (Saheb & Jog, 1999). However, natural materials cannot provide all the desirable in industry properties because of biodegradability and poor resistance to moisture or ultraviolet; therefore, synthetic composites are of great interest (Beloshenko, Voznyak, Voznyak, & Savchenko, 2017). An advantage of soft (nonlinear) composites over hard (linear) composites is high elasticity, allowing significant reversible geometry changes (including buckling Andrianov, Kalamkarov, & Weichert, 2012; Gao & Li, 2017; Li, Kaynia, Rudykh, & Boyce, 2013; Parnes & Chiskis, 2002); consequently, their effective properties can be tuned by elastic deformation. In particular, elastic waves in soft composites can be controlled by deformation (Bertoldi & Boyce, 2008; Galich, Fang, Boyce, & Rudykh, 2017; Galich & Rudykh, 2017; Slesarenko, Galich, & Rudykh, 2018; Nam, Merodio, Ogden, & Vinh, 2016). It is worth noting also that many soft biological tissues are found to possess fiber-matrix microstructures (Humphrey, 2002), and the soft tissues frequently experience large deformations due to different physiological processes. Hence, investigation of elastic wave propagation and instabilities (that can significantly influence

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elastic waves) in 3D fiber-reinforced composites undergoing finite deformations can be beneficial for biomedical applications such as ultrasound testing.

By employing the nonlinear elastic theory (Truesdell & Noll, 1965) and a phenomenological approach, Scott and Hayes (1976) considered small amplitude plane waves superimposed on a homogeneous deformation in the so-called idealized fiber-reinforced materials, assuming incompressibility of the matrix and inextensibility of the fibers. Later, Scott (1991, 1992), and Rogerson and Scott (1994) extended this analysis for a broader class of finitely deformed fiber-reinforced materials with marginally extensible and compressible constituents. Recently, Ogden and Singh (2011) exploited a phenomenological theory of invariants and presented a more general and transparent formulation of the theory for small amplitude waves propagating in deformed transversely isotropic hyperelastic solids. This approach was successfully utilized for calculating speeds of homogeneous plane waves (Vinh & Merodio, 2013) and non-principal Rayleigh waves (Nam, Merodio, & Vinh, 2016; Vinh, Merodio, Hue, & Nam, 2014) in various deformed transversely isotropic incompressible solids. More recently, Galich, Slesarenko, and Rudykh (2017) employed a micromechanics based approach (deBotton, Hariton, & Socolsky, 2006) and derived explicit closed form expressions for phase and group velocities of shear waves propagating in finitely deformed 3D FCs. By application of the Bloch–Floquet approach in the finite element code (Aberg & Gudmundson, 1997; Bertoldi & Boyce, 2008), Galich, Slesarenko et al. (2017) also investigated dispersion of shear waves propagating along the fibers in finitely deformed FCs with square arrays of fibers. However, the influence of fiber arrangement on shear wave propagation in finitely deformed 3D FCs has not been investigated.

In this paper, we investigate the influence of fiber arrangement on small amplitude shear wave propagation in finitely deformed FCs with rectangular arrays of cylindrical fibers. We limit our consideration of the applied finite deformation prior to the onset of elastic instabilities. Recall that elastic instabilities in hyperelastic FCs may develop according to the microscopic or macroscopic scenarios depending on the material composition (Slesarenko & Rudykh, 2017; Triantafyllidis & Maker, 1985). In particular, the conditions for the onset of macroscopic instabilities, characterized by wavelengths significantly larger than the characteristic microstructure size, can be predicted by evaluation of the homogenized tensor of elastic moduli that can be obtained from phenomenological (Merodio & Ogden, 2002; Merodio & Pence, 2001; Qiu & Pence, 1997) or micromechanics based (Agoras, Lopez-Pamies, & Castañeda, 2009; Lopez-Pamies & Castañeda, 2006a; 2006b; Rudykh & deBotton, 2012) material models for transversely isotropic FCs. The conditions for the onset of microscopic instabilities, characterized by wavelengths comparable with the characteristic microstructure size, can be predicted by the Bloch–Floquet analysis (Geymonat, Müller, & Triantafyllidis, 1993; Michel, Lopez-Pamies, Ponte Castañeda, & Triantafyllidis, 2010; Nestorovic & Triantafyllidis, 2004; Slesarenko & Rudykh, 2017; Triantafyllidis, Nestorovic, & Schraad, 2006). However, for uniaxially compressed 3D FCs with rectangular arrays of fibers, dependence of the critical stretches and wavelengths on fiber arrangement has not been discussed in literature. Here, we specifically focus on the influence of fiber arrangement, and we identify the conditions leading to the macroscopic/microscopic instabilities in hyperelastic FCs with rectangular arrays subjected to compressive deformations along the fibers. We show that the critical stretches for FCs with rectangular arrays of fibers are bounded by the corresponding critical values for the 3D laminates and FCs with square arrays of fibers. Remarkably, this holds true regardless of the volume fraction of fibers and contrast in shear moduli between the fibers and matrix. Next, we study the dispersion curves of shear waves in FCs deformed close to the instability point, where the most remarkable behavior of shear waves is observed. We observe that the polarization vectors of long shear waves propagating along the fibers in FCs with large volume fractions of fibers (e.g. 25%) rotate upon attaining the critical deformation level. These changes in direction of polarization indicate that fibers develop buckling shapes in non-principal planes. Moreover, polarizations of the lowest frequency shear waves in FCs with small fiber volume fractions (e.g. 5%) change their directions depending on the aspect ratio of the FC arrays. Thus, the buckling shapes can be tailored through the choice of the FC array aspect ratio.

2. Theoretical background

Consider a continuum body and identify each point in the reference (undeformed) configuration with vector $\mathbf{X}$. In the current (deformed) configuration, the new location of the corresponding points is defined by vector $\mathbf{x} = \mathbf{x}(\mathbf{X}, t)$. Then, the deformation gradient is $\mathbf{F} = \partial \mathbf{x} / \partial \mathbf{X}$, and $J \equiv \det \mathbf{F} > 0$. For a hyperelastic compressible material with a strain energy function $\psi(\mathbf{F})$, the first Piola–Kirchhoff stress tensor can be calculated as follows

$$\mathbf{P} = \frac{\partial \psi(\mathbf{F})}{\partial \mathbf{F}}. \tag{1}$$

For an incompressible material, $J = 1$ and Eq. (1) modifies as

$$\mathbf{P} = \frac{\partial \psi(\mathbf{F})}{\partial \mathbf{F}} - p\mathbf{F}^{-T}, \tag{2}$$

where p represents an unknown Lagrange multiplier. The corresponding Cauchy stress tensor is related to the first Piola–Kirchhoff stress tensor via the relation $\boldsymbol{\sigma} = J^{-1}\mathbf{P} \cdot \mathbf{F}^T$.

In the absence of body forces the equations of motion can be written in the undeformed configuration as

$$\mathrm{Div}\mathbf{P} = \rho_0 \frac{D^2\mathbf{x}}{Dt^2}, \tag{3}$$