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Bistability criterion for electrostatically actuated initially curved micro plates

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ABSTRACT

The criterion defining the geometric parameters guaranteeing bistable behavior of electrostatically actuated curved axisymmetric circular plate is established. The usage of Berger's approximation for von-Kármán nonlinear plates, combined with single degree of freedom (DOF) reduced order (RO) modeling, allowed derivation of a simple semi-analytical bistability criterion, obtained in the form of an implicit algebraic equation in terms of critical deflection and plate geometric parameters. The criterion is verified by direct numerical solutions, combined with the arc-length method. Case studies are presented, illustrating the implementation of the suggested criterion as a useful tool for the early design stage for MEMS/NEMS devices.

1. Introduction

Bistability of a structural element, enabling transition between two coexisting stable configurations, is an attractive feature, which can be utilized for various applications in the realm of micro- and nanoelectromechanical systems (MEMS/NEMS) (Hu and Burgueo (2015). Examples range from energy harvesters (Arrieta, Hagedorn, Erturk, & Inman, 2010; Harne & Wang, 2013; Zhu & Zu, 2013), to transducers (Akkas & Odeh, 2001), sensors (Harne & Wang, 2014), switches (Intaraprasonk & Fan, 2011), non-volatile memories (Charlot, Sun, Yamashita, Fujita, & Toshiyoshi, 2008) and micro-pumps (Liu, 2010; Machauf, Nemirovsky, & Dinnar, 2005; Nisar, Afzulpurkar, Mahaisavariya, & Tuantranont, 2008; Pan, Ng, Liu, Lam, & Jiang, 2001; Tavakol & Holmes, 2016; Wagner, Quenzer, Hoerschelmann, Lisec, & Juerss, 1996). These structures are often actuated using electrostatic forces, allowing design of low power consumption, short response time devices, conveniently compatible with the current fabrication processes and driving electronic circuity.

While theoretical and numerical analysis, as well as practical implementation of 1-D beam-type structures, was extensively explored (Das & Batra, 2009; Medina, Gilat, & Krylov, 2014b; 2016b; Ouakad & Younis, 2010; Pane & Asano, 2008; Saif, 2000; Zhang, Yan, Peng, & Meng, 2014), much less attention was paid to the behavior of 2-D and 3-D bistable micro structures, such as plates and shells. Within the theoretical arena, the unabatedly expanding research on the behavior of electrostatically actuated 2-D structural elements was initially devoted to the behavior of electrostatically actuated micro scale membranes and flat plates (Batra, Porfiri, & Spinello, 2008a; 2008b; Nayfeh, Younis, & Abdel-Rahman, 2005; Pelesko & Chen, 2003; Vogl & Nayfeh, 2005; Wang, Lin, Li, & Feng, 2011; Zhao, Abdel-Rahman, & Nayfeh, 2004). Recently, this research has expanded into the electrostatically operated, slightly curved micro plates and shallow shells. The dynamic

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(Ghayesh, Farokhi, & Alici, 2016) and static (Jallouli et al., 2016; Saghir, Bellaredj, Ramini, & Younis, 2016; Saghir & Younis, 2016) responses were examined analytically, using a combination of reduced order (RO) modeling and perturbation methods, as well as numerically, by means of finite elements (FE) and finite differences (FD) analyses. In addition, several experimental works reported on the response of slightly curved micro plates to electrostatic actuation (Ghayesh et al., 2016; Jallouli et al., 2016; Krysko, Awrejcewicz, Zakharova, Papkova, & Krysko, 2018; Saghir et al., 2016; Saghir & Younis, 2018). In all of these works, the non-flat geometry was considered to be a result of imperfections, which are liable to appear during the fabrication process, and are too small to evoke bistability, and as such insufficient to lead to snap-through collapse. In contrast to these works, Seffen and Vidoli (2016) and Loukaides, Smoukov, and Seffen (2014) have studied the behavior of a magnetically actuated magnetic spherical cup, having a high enough curvature to generate a bistable response. However, in these works, the actuation was induced by a magnet located at a large distance from the shell, making any nonlinear effects negligible. The interplay between a nonlinear electrostatic force and the geometric parameters of an initially curved plate, was studied in Medina, Gilat, and Krylov (2016a). It was theoretically shown that electrostatically actuated circular initially curved plates may exhibit both pull-in and snap-through instabilities. The analysis was based on a reduced order (RO) model of Föppl von-Kármán plate formulation, detailing the plate axisymmetric response, ruled by two equilibrium equations. In a later work, Medina, Gilat, and Krylov (2017), a simple RO model of electrostatically actuated curved circular microplates, based on the Berger plate approximation, was suggested and examined. In Sobota and Seffen (2017) the effects of various boundary conditions on the bistable behavior of axisymmetrical shallow shells, was studied using polynomial displacement fields.

From a practical point of view, incorporation of bistable 2-D structures in real MEMS devices is very rare. We can argue that one of the reasons for this, apart from the difficulty in micro fabrication of non-planar structures, is the lack of appropriate tools for the design of such elements. Probably the main question which arises at the first stage of the bistable device design is “What are the geometric parameters and operational conditions guaranteeing bistability in such a device?” In contrast to beams, where simple lumped models and bistability criteria (Krylov, Ilic, Schreiber, Seretensky, & Craighead, 2008; Medina et al., 2014b; Simitses & Hodges, 2006) are routinely used at the early stages of the design in order to establish the needed parameters, design of 2-D structures is much more challenging. It may require the use of advanced nonlinear coupled FE analysis, or multi DOF RO models. The implementation of these models is time consuming, impose demanding qualification requirements to the MEMS designer, burdening the overall design process. The modeling of these structures may also be complicated by the presence of bifurcations, increasing the sensitivity of the structure overall response to minute changes in any one of its parameters (Medina, Gilat, Ilic, & Krylov, 2014a; Medina, Gilat, & Krylov, 2016b).

In the present work, Berger’s RO (BRO) model is employed to establish a simple semi-analytical criterion, defining the geometric parameters guaranteeing bistability for electrostatically actuated initially curved circular microplates. Despite that the widely used von-Kármán theory can be exploited for this purpose, the usage of the Berger model allows analytical derivation of an expression obtained in the form of an implicit algebraic equation, relating the critical deflection, the initial elevation and the thickness of the plate, as well as the distance between the plate and the electrode. The equation is then solved numerically to obtain the bistability criterion. Rigorous numerical validation was then carried out against von-Kármán RO (vKRO) axisymmetric based models and a full scale 3D FE model, which does not use the axisymmetry assumption. The paper concludes with dimensional examples, illustrating the implementation of the dervied condition. It is shown that the suggested criterion can be used by researchers and MEMS/NEMS practitioners alike, as an efficient tool for the design of bistable micro and nano structures.

2. Formulation

Consider a shallow, initially curved, circular micro plate, as shown in Fig. 1. The plate has thickness $\hat{d}$, which is small compared to its radius R. The plate is assumed to be made of homogeneous isotropic, linearly elastic material, with Young’s modulus E, and Poisson’s ratio ν. The initial, as-designed, shape of the stress-free plate is described by the function $\hat{w}_0(\hat{r},\theta) = \hat{h}_0 z_0(\hat{r},\theta)$, where $\hat{h}_0$ is the elevation of the plate central point above its edge, and $z_0(\hat{r},\theta)$ is a non-dimensional function such that $\max_{\hat{r}\in[0,R]}\left[z_0(\hat{r},\theta)\right] = 1$. The plate is actuated by transverse distributed electrostatic force, generated by a planar electrode located at a distance $\hat{g}_0$ (the gap) from the plate circumferential boundary.

In an earlier work (Medina et al., 2017), it was shown that Berger’s approximation can be suitable and convenient to simplify the formulation describing the behavior of curved shallow plates subjected to electrostatic loads. Consequently, in this work we describe the plate using Berger’s formulation, expressed by the following dimensional integro-differential equation (Berger, 1954; Medina et al., 2017; Villagio, 1997)

$$\hat{\nabla}^4(\hat{w}-\hat{w}_0) - \frac{12}{\hat{d}^2}\left(\int_0^R\left(\left(\frac{d\hat{w}}{d\hat{r}}\right)^2 - \left(\frac{d\hat{w}_0}{d\hat{r}}\right)^2\right)\hat{r}\,d\hat{r}\right)\hat{\nabla}^2\hat{w} = -\frac{\hat{f}^e}{D} \tag{1}$$

with the following homogenous boundary conditions

$$\hat{w} = 0, \quad \frac{d\hat{w}}{d\hat{r}} = 0 \quad @ \quad \hat{r} = R \tag{2}$$