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Constitutive boundary conditions for nonlocal strain gradient elastic nano-beams

Raffaele Barretta*, Francesco Marotti de Sciarra

Department of Structures for Engineering and Architecture, University of Naples Federico II, via Claudio 21, Naples 80125, Italy

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ABSTRACT

Nonlocal strain gradient integral model of elasticity, extension of the fully nonlocal integral law, is widely adopted to assess size effects in nano-beams. The bending moment is sum of convolutions of elastic curvature and of its derivative with a smoothing kernel. For nanomechanical problems on unbounded domains, such as in wave propagation, the nonlocal strain gradient integral relation is equivalent to a differential law with constitutive conditions of vanishing at infinity. For bounded nano-beams, the constitutive boundary conditions (CBC) must be added to close the constitutive model. The formulation of these CBC is an original contribution of the paper. Equivalence between nonlocal strain gradient integral model of elasticity and the differential problem with CBC is proven. It is shown that the CBC do not conflict with equilibrium and provide a viable approach to study size-dependent phenomena in nano-beams of applicative interest. Theoretical outcomes are illustrated by examining the static scheme of a nano-actuator modelled by a nano-cantilever inflected by an end-point load. The relevance of a proper formulation of boundary conditions is elucidated by comparing the numerical results with previous attempts in literature.

1. Introduction

Modeling and assessment of size effects in nanomaterials is a subject of wide and current interest in the community of Engineering Science. The realization of ground-breaking nano-sensors and nano-actuators, as basic structural elements of scanners, mirrors, gyroscopes, springs and many similar other nanoscale systems, is an important target with countless conceivable applications (see e.g. Baghani, 2012; Farokhi & Ghayesh, 2018; Farokhi, Ghayesh, & Gholipour, 2017; Ghayesh, Farokhi, & Amabili, 2013; Medina, Gilat, & Krylov, 2017; Mojahedi, 2017; Sedighi, 2014; Sedighi & Bozorgmehri, 2016; Sedighi, Daneshmand, & Abadyan, 2016). Nano-materials are exploited also as excellent components for reinforcement in nano-structures (see e.g. Acierno, Barretta, Luciano, Marotti de Sciarra, & Russo, 2017; Govorov, Wentzel, Miller, Kanaan, & Sevostianov, 2018; Hashemi, 2016; Patti et al., 2015).

Size-dependent behavior of continua is conveniently described by structural mechanics. Several nonlocal constitutive laws have been proposed in literature and are being extensively investigated as summarized below.

Two different definitions of fully nonlocal elasticity exist for a continuum.

* Corresponding author.
E-mail addresses: rabarret@unina.it (R. Barretta), marotti@unina.it (F. Marotti de Sciarra).

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The first is a strain-driven formulation that was introduced by Eringen (1983). It provides the nonlocal stress field as output of the integral convolution of elastic strain with a smoothing kernel. This model, successfully applied by Eringen (1983) to nonlocal problems defined on unbounded domains, cannot be adopted to assess size-effects in nanomechanics (Romano, Barretta, Diaco, & Marotti de Sciarra, 2017a). This conclusion, acknowledged in literature (see e.g. Barati, 2017; Faghidian, 2018a; 2018b; Fathi & Ghassemi, 2017; Fernández-Sáez & Zaera, 2017; Karami, Shahsavari, Janghorban, & Li, 2018; Sahmani & Aghdam, 2018; Vila, Fernández-Sáez, & Zaera, 2017; Xu, Zheng, & Wang, 2017c; Zhang, 2017; Zhu & Li, 2017), is motivated from the fact that, for bounded continuous structures, the constitutive boundary conditions associated with Eringen's integral law conflict with equilibrium requirements (Barretta et al., 2018a; Romano & Barretta, 2016).

The second way to formulate a fully nonlocal elastic model was proposed by Romano and Barretta (2017a), is of stress-driven type and consists in defining the nonlocal elastic strain field as output of the integral convolution between stress field and a smoothing kernel. Such a model leads to well-posed elastostatic problems also for structures defined on bounded domains.

Strain-driven and stress-driven fully nonlocal formulations for nano-beams were compared by Romano and Barretta (2017b). Integral nonlocal laws were adopted in several static, dynamic and thermoelastic structural problems of nanotechnological interest (Apuzzo, Barretta, Luciano, Marotti deSciarra, & Penna, 2017; Barretta, Faghidian, & Luciano, 2018d; Barretta, Luciano, Marotti de Sciarra, & Ruta, 2018e; Barretta et al., 2018b; Barretta, Čanadija, Luciano, & Marotti de Sciarra, 2018c; Fernández-Sáez, Zaera, Loya, & Reddy, 2016; Mahmoudpour, Hosseini-Hashemi, & Faghidian, 2018; Oskouie, Ansari, & Rouhi, 2018).

The fully nonlocal elastic integral relations were also convexly combined with the local elastic laws by Eringen (1972, 1987) and by Romano, Barretta, and Diaco (2017b), formulating thus strain-driven and stress-driven two-phase mixtures for nano-engineering problems (Barretta, Fabbrocino, Luciano, & Marotti de Sciarra, 2018f; Barretta, Faghidian, Luciano, Medaglia, & Penna, 2018g; Fernández-Sáez & Zaera, 2017; Pisano & Fuschi, 2003; Wang, Zhu, & Dai, 2016; Zhu, Wang, & Dai, 2017). Advantages, basic properties, singular behaviors and boundary effects of most nonlocal integral models of literature are studied by Romano, Luciano, Barretta, and Diaco (2018).

Eringen's integral law (1983) was also combined with strain gradient elasticity by Lim, Zhang, and Reddy (2015) to formulate a higher-order nonlocal theory. According to this approach, the stress is defined as sum of two integral terms. The first one is Eringen's convolution between elastic strain and a smoothing kernel depending on a nonlocal parameter. The second one, involving also a gradient parameter, is the derivative of the convolution of the elastic strain gradient with a smoothing kernel depending on a nonlocal parameter.

For inflected nano-beams the nonlocal strain gradient model is expressed by Eq. (1), where stress and elastic strain fields are respectively bending moment and elastic flexural curvature fields. In agreement with the methodology adopted by Eringen (1983), Eq. (1) (equipped with the special bi-exponential kernel Eq. (3)) was replaced by Lim et al. (2015) with the differential law Eq. (18) to study wave propagations.

The differential law Eq. (18) is considered in literature to be equivalent to the integral relation Eq. (1) also for structural problems defined on bounded domains. The elastostatic problem of a Bernoulli-Euler inflected nano-beam, associated with the differential law Eq. (18), is governed by a differential equation of higher-order than of the one of the classical local problem. Accordingly, additional non-classical higher-order boundary conditions must be selected to close the nonlocal strain gradient elastostatic problem. In all treatments, this issue is addressed by prescribing additional higher-order boundary conditions. Two different usual choices consist in imposing kinematic or static (higher-order) boundary conditions of strain gradient theory (see e.g. Li, Li, and Hu, 2016, Eq.(60)) and (see e.g. Xu, Wang, Zheng, and Ma, 2017b, Eq.$(31)_5$).

The ensuing structural responses are notably influenced by these choices.

This issue, widely debated in literature, is considered to be an open question of Engineering Science (see e.g. Xu, Zhou, & Zheng, 2017a).

A definite response is provided in Proposition 3.1 of this paper.

The non-classical boundary conditions to be imposed to solve the elastostatic problem of a nonlocal strain gradient nano-beam are given by Eq. (19). They are naturally detectable by the nonlocal strain gradient integral law Eq. (1) and therefore are of constitutive kind. Unlike the fully nonlocal integral relation, the nonlocal strain gradient law Eq. (1) leads to well-posed elastostatic problems also for bounded nano-structures.

Importantly: since the corresponding displacement solutions exhibit softening and stiffening structural behaviors for increasing nonlocal and gradient parameters respectively, the nonlocal strain gradient law provides an effective approach to model significantly a wide class of nano-engineering problems. Also, it is worth noting that in wave propagation (Lim et al., 2015), which refer to unbounded domains, the constitutive boundary conditions (CBC) Eq. (19) are verified since the involved fields rapidly vanish at infinity.

Moreover, when the gradient parameter l is assumed to vanish, CBC Eq. (19) coincide with the ones of Eringen's fully nonlocal integral model established in (Romano et al. 2017a, Prop. 3.1, Eq.(5)).

Effectiveness of the theoretical results in Proposition 3.1 is illustrated in Section 5, by examining a cantilever nano-beam inflected by an end-point load.

Numerical evidences reveal substantial differences from the technical standpoint between the contributed results (associated with the model in Propositon 3.1) and the ones of literature.

The present paper provides a significant advancement in the theory of nonlocal strain gradient elasticity and an effective methodology for structural modelling and design of modern nano-devices of nanotechnological interest.