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Structural discontinuity as generalized strain and Fourier transform for discrete-continuous systems

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ABSTRACT

We consider a segmented structure, possibly connected with a continuous medium, as initially homogeneous, where discontinuities arise as localized strains induced by self-equilibrated localized actions. Under this formulation augmented by interface conditions, the linearized formulation remains valid. This approach eliminates the need for examining separate sections with subsequent conjugation. Only conditions related to the discontinuities should be satisfied, while the continuity in other respects preserves itself automatically. No obstacle remains for the continuous Fourier transform. For a uniform partitioning, the discrete transform is used together with the continuous one. We demonstrate the technique by obtaining the Floquet wave dispersive relations with their dependence upon interface stiffness. To this end, we briefly consider the flexural wave in the segmented beam on Winkler's foundation, the gravity wave in a plate (also segmented) on deep water and the Floquet–Rayleigh wave in such a plate on an elastic half-space. Besides, we present the wave equations developed for an elastic medium with discontinuities.

1. Introduction

In this paper, we show how the continuous Fourier transform, an effective method in the linear analysis commonly used for a system homogeneous in the respective coordinate, can be extended to a segmented structure possibly interacting with a continuous medium.

Let a homogeneous system have point contacts with some other structures. For example, it could be an elastic beam supported by deformable rods. We can carry out the Fourier transform with accounting for the (unknown) contact forces and obtaining then the latter based on respective force-displacement relations. Note that in the case of a distributed contact, we end up with an integral equation on the corresponding domain. In the case of discrete supports distributed uniformly, the discrete Fourier transform leads to a single ratio. Gueorguiev, Gregory McDaniel, Dupont, and Felsen (2000) examined such a system as a waveguide with several periodical arrays of attachments.

It may seem less evident that similar considerations are applicable not only for the contact inhomogeneities but local structural discontinuities as well. Note that structures with discontinuities present in different areas. An array of surface cracks in a material (in this connection, see Joglekar & Mitran, 2016) and a cracked sea ice cover can serve as examples. Segmented constructions can also be mentioned such as stationary or pontoon bridges. Note that in the ice cover case, in-plane compressive forces and the crack closure effect strongly affect the 'effective' hinge stiffness manifested in angle

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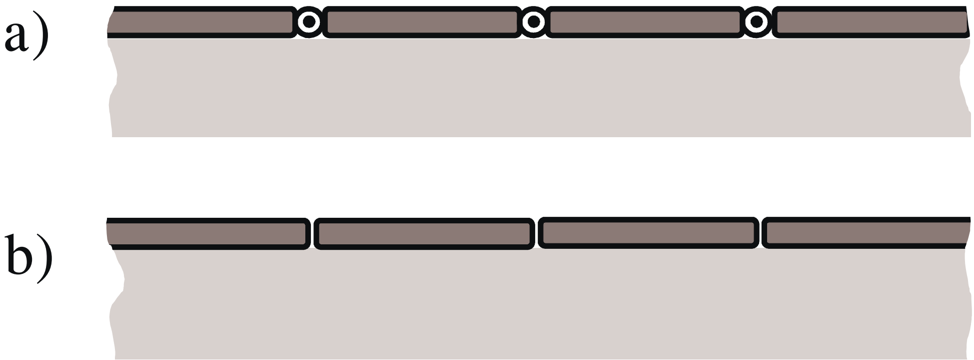

Fig. 1. The segmented elastic beam in contact with a 2D medium homogeneous in the longitudinal coordinate. The discontinuities allowing rotations under the bending moment (a) and entirely separated parts of the beam, which can interact with each other through the medium (b) - as Professor Gennady Mishuris sees it.

interactions of the adjacent sections. In this connection, see Rice and Levy (1972), Dempsey, Adamson, and DeFranco (1995), Slepyan, Dempsey, and Shekhtman (1995). (The in-plane forces also play a nontrivial role in the analysis developed below, see Section 2.2.)

We consider an analytical technique that permits the Fourier integral transform in studying the static deformations, oscillations, and waves in discrete-continuous structures. As illustrative examples, we examine a homogeneous elastic beam (or a plate) divided into sections and coupled with a continuous medium. We show two possible configurations schematically in Fig. 1 (the parts can rotate and shift transversely relative to each other under the bending moment and transverse force, respectively).

For the analytical study of such systems we use the approach earlier discussed in fracture mechanics where a crack was represented as the result of the action of generalized forces on the continuous medium, (Slepyan, 1981, Section 1.4; 2002, Section 5.7)

We use Lagrangian formulation considering the discontinuities as generalized strains created in an initially homogeneous structure by localized, self-equilibrated, generalized loads. Mathematically, this approach differs by using some higher order 'deltas,' the derivatives of the Dirac delta-function, higher than those related to real external localized forces.

For example, the functions $\delta(x)$ and $\delta'(x) = \mathrm{d}\delta(x)/\mathrm{d}x$, corresponding to jump discontinuities in the transverse force and bending moment in the beam, make the internal force and moment discontinuous. These facts are well-known. At the same time, no one prevents us from continuing with the derivatives of higher orders taking generalized loads $\delta''(x)$ and $\delta'{}'(x)$. The latter corresponds to the structural discontinuities, the jumps in the inclination angle of the beam $w'(x)$ and its displacement $w(x)$, respectively. Concerning the above considerations, we note that the delta-function can be defined as the derivative of a unite jump of discontinuity (see the related topics, e.g., in Bremermann, 1965).

Formally, the 'loads' creating structural discontinuities do not affect the regular strain outside of the singular points but result in supersingular strain at these points. The latter, however, have no connection with the actual interface interaction, for which we introduce relations between the discontinuities and the corresponding force factors independently.

In doing so, we eliminate the obstacle for the use of the integral Fourier transform for the segmented structure with an adjoining medium, which reflects itself by Green's function. In the case of a uniform discrete distribution of the discontinuities, the coupled discrete and continuous transforms present in the analysis.

As a simple example consider the static problem for an elastic beam rested on Winkler's foundation.

1.1. Elastic beam with a single hinge

The Euler-Bernoulli equation for a uniform elastic beam on Winkler's foundation is

$$D\frac{\mathrm{d}^4 w(x)}{\mathrm{d}x^4} + \varkappa w(x) = q(x)\,, \quad -\infty < x < \infty\,, \tag{1}$$

where $w(x)$ is the displacement, D is the bending stiffness, $\varkappa$ is the bed stiffness and $q(x)$ is the external load.

Let there be an integrity violation at $x = 0$, such that results in a jump discontinuity of the inclination angle (the displacement derivative). Denote it by $\alpha = w'(+0) - w'(x-0)$.

We assume that there exists a linear relation between the angle discontinuity and the bending moment

$$\mathcal{M}(\pm 0) = \mathcal{M}(0) = -\varkappa_M \alpha\,, \quad \varkappa_M = \mathrm{const} \ge 0\,. \tag{2}$$

In particular, the entire separation with respect to the rotation corresponds to $\mathcal{M}(\pm 0) = 0\,(\varkappa_M = 0)$, whereas $\alpha = 0\,(\varkappa_M = \infty)$ corresponds to the intact beam without the discontinuity.