International Journal of Engineering Science 129 (2018) 84–95

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Nonlinear mechanics of nanoscale tubes via nonlocal strain gradient theory

Mergen H. Ghayesh*, Ali Farajpour

School of Mechanical Engineering, University of Adelaide, South Australia 5005, Australia

ARTICLE INFO

Article history:
Received 19 March 2018
Accepted 15 April 2018

Keywords:
Nanotubes
Nonlocal strain gradient model
Nonlinear mechanics
Galerkin scheme

ABSTRACT

A size-dependent nonlinear nonlocal strain gradient model for nanoscale tubes is proposed in this investigation and the forced mechanical behaviour is examined. This continuum model is better capable of incorporating size effects as it includes two independent length-scale parameters. The scale-dependent elastic energy and motion energy as well as the work carried out by the excitation load are formulated. The non-classical nonlinear differential equation of motion of the nanoscale tube is obtained using Hamilton's work/energy principle together with the nonlocal strain gradient elasticity. A precise numerical solution is presented for the nonlinear dynamic characteristics within the framework of Galerkin's scheme in conjunction with a continuation approach. The influences of nanosystem parameters such as the scale parameters, the length-to-gyration-radius ratio as well as the amplitude of the excitation force on the frequency/force responses are explored and discussed in details.

1. Introduction

In many nanoelectromechanical systems (NEMS) such as nanoelectromechanical sensors, nanoscale resonators and energy harvesting nanodevices, understanding the vibration response of nanoscale structural elements under external loading is of great importance as these NEMS devices usually work based on vibrating nanostructures (Attia, 2017; Attia & Abdel Rahman, 2018; Bakhshi Khaniki & Hosseini-Hashemi, 2017; Ebrahimi & Barati, 2016; Ebrahimi, Barati, & Dabbagh, 2016; Hashemi, 2016; Kiani, 2016; Lu, Guo,& Zhao, 2017; Nejad & Hadi, 2016; Shahverdi & Barati, 2017; She, Yuan, Ren, & Xiao, 2017; Şimşek, 2016; SoltanRezaee & Afrashi, 2016). This motivates many researchers to develop accurate continuum models in order to extract the vibration characteristics of nanoscale beams, rods and plates. A precise continuum model for a nanoscale structure should be a function of its size since the small scale influence cannot be ignored at such ultrasmall levels.

Different size-dependent theoretical models including the nonlocal version of the continuum mechanics (Arash & Wang, 2012; Aydogdu, 2015; Farajpour, Rastgoo, & Farajpour, 2017), the modified theory of the couple stress elasticity (Farokhi & Ghayesh, 2015; 2018; Ghayesh & Farokhi, 2015b; Ghayesh, Farokhi, Gholipour, & Tavallaeinejad, 2018; Ghayesh, Amabili, & Farokhi, 2013b; Ghayesh, Farokhi, & Amabili, 2013b; 2014; Gholipour, Farokhi, & Ghayesh, 2015), and a strain gradient theory (Ghayesh, Amabili, & Farokhi, 2013a) have recently been proposed in the open literature. All these valuable size-dependent theories include only one length-scale parameter to predict the mechanics of structures at nanoscale levels.

* Corresponding author.
E-mail address: mergen.ghayesh@adelaide.edu.au (M.H. Ghayesh).

https://doi.org/10.1016/j.ijengsci.2018.04.003

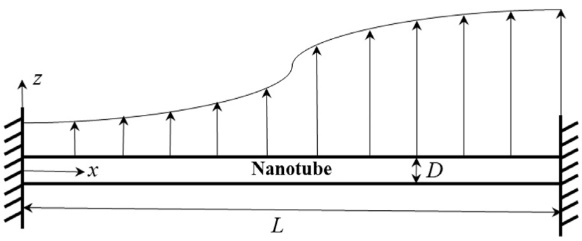

Fig. 1. A representation of a nanoscale tube under external excitation force along the z direction.

However, recently, a new higher-order scale-dependent model with two different scale parameters has been introduced by Lim, Zhang, and Reddy (2015) on the basis of the nonlocal strain gradient theory (NSGT). They also used this theory for some wave propagation problems at the nanoscale level. Li and Hu (2015) investigated the buckling of nanobeams via the help of the NSGT and the Euler–Bernoulli beam theory (EBBT). Furthermore, Şimşek (2016) developed a scale-dependent mathematical framework to investigate the large amplitude free vibration of functionally graded (FG) nanoscale beams employing the NSGT and a Hamiltonian method. Zhu and Li (2017) presented an exact solution for nanorods under axial loading based on the nonlocal strain gradient model. They also carried out molecular dynamics (MD) simulations on this problem, and found a great agreement between the scale-dependent results of the NSGT and those of MD simulations. In addition, a bi-Helmholtz size-dependent model was developed by Barati and Zenkour (2017) for the wave propagation analysis of a nanoscale system made of two porous beams based on the NSGT. Moreover, the applications of the nonlocal strain gradient continuum model to piezoelectric nanoplate-based electromechanical sensors (Farajpour, Rastgoo, Farajpour, & Mohammadi, 2016), the wave propagation in nanostructures (Ebrahimi et al., 2016; Li, Hu, & Ling, 2016), and the buckling analysis of protein microtubule bundles in living cells (Farajpour & Rastgoo, 2017) have been reported. Lu, Guo, and Zhao (2017) carried out a theoretical analysis to understand the linear free vibration of nanobeams with simply supported edges via the Timoshenko theory of beams in conjunction with the NSGT.

All of the above-mentioned valuable articles are limited to either linear vibration problems or the nonlinear free vibration of nanoscale beams. In addition, the nonlinear vibration problems were studied by considering only one or two shape functions for the displacement components which are not enough in order to obtain an accurate solution. The aim of the present work is to present a size-dependent theoretical formulation to predict the nonlinear forced vibration behaviour of nanotubes under external transverse loading based on a combination of the nonlocal continuum mechanics and the strain gradient elasticity; this is for the first time. The proposed continuum model contains two different length-scale parameters for incorporating size effects. The EBBT and NSGT in conjunction with Hamilton's principle are used to obtain the nonlocal strain gradient differential equation of motion of the nanoscale tube. Galerkin's method and a numerical technique within the framework of the continuation method are employed to extract the nonlinear dynamic characteristics of the nanosystem. It is found that both length-scale parameters have a crucial role to play in the nonlinear frequency-response of nanotubes.

2. A nonlinear nonlocal strain gradient nanobeam model

In this section, a nonlinear non-classical beam model is proposed for the forced vibration of ultrasmall tubes under external harmonic loading. First, based on the nonlinear EBBT and the NSGT, the elastic energy, motion energy and the work done by the external force are calculated. After that, the nonlinear nonlocal strain gradient differential equation of motion of the nanosystem is derived. Finally, the scale-dependent vibration characteristics of the nanotube are obtained by an accurate numerical technique.

Fig. 1 depicts a nanoscale tube with length L and outer diameter D subject to an external harmonic load along the transverse direction. As can be seen from this figure, a Cartesian coordinate frame with axes x and z is employed to indicate the position of each point. The cross-sectional area and the moment of inertia of the nanotube are indicated by A and I, respectively. The external harmonic force is $F(x)\cos(\omega t)$ where $F(x)$ denotes the amplitude of the external load, and ω is its frequency.

Based on the EBBT, the nonlinear strain-displacement relationship for nanoscale tubes is (Ghayesh & Amabili, 2014; Ghayesh, Farokhi, & Gholipour, 2017b; Ghayesh, Farokhi, & Hussain, 2016)

$$\varepsilon_{xx}(x,z,t) = \frac{\partial u(x,t)}{\partial x} + \frac{1}{2}\left(\frac{\partial w(x,t)}{\partial x}\right)^2 - z\frac{\partial^2 w(x,t)}{\partial x^2}, \tag{1}$$

where ε_{xx} and u stand for the axial strain and displacement, respectively; also, w and t are, respectively, the transverse displacement and time. The couple resultant (M_{xx}) and the force resultant (N_{xx}) are given by

$$M_{xx} = \int_A z t_{xx} dA, \; N_{xx} = \int_A t_{xx} dA. \tag{2}$$