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Three-dimensional singular elastostatic fields in a cracked Neo-Hookean hyperelastic solid

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ABSTRACT

The boundary-value problem of Neo-Hookean incompressible hyperelastic cracked solid under a superposition of a plane deformation to an anti-plane one is formulated. An asymptotic analysis is then employed to compute the elastostatic fields near the crack front and their principal properties are illustrated. In a particular basis, the crack is bound to open independently of the magnitude and the mode of the boundary conditions at infinity. The stress field components have different singularities and each one can posses more than one singular term. Some disagreements with the linear theory are evidenced.

1. Introduction

In fracture mechanics, the three-dimensional character nature of the deformation and stress fields induced by a crack is well recognized (Rice, 1968). This is asserted by common experimental observation (Rosakis & Ravi-Chandar, 1986) and numerical experiences (Nakamura & Parks, 1988; 1990; Rannou et al., 2010). However, only a few theoretical analyses are done to assess analytically the three-dimensional elastostatic fields, especially for nonlinear behaviours. In fact, the boundary value mathematical problem associated with the three-dimensional crack geometry is rarely easy to solve analytically (Ogden, 1997). This is due to the complicated crack geometry which can be modelled as a conical point, a front, a vertex or the intersection of a front with a vertex (Yosibash, 2011).

To this end, the objective of this paper is focused in the investigation and the analysis of the elastostatic fields corresponding to the superposition of the in-plane transformation to the anti-plane transformation in a cracked Neo-hookean hyperelastic long solid. This is a particular class of the three-dimensional crack problem with a simple hyperelastic potential which can elucidate the other more complicated class of problem. We mention here that from a mathematical point of view the neoHookean material is of prime interest for finite elasticity to deduce analytic solutions (Hill, 2001) but this model is poor to predict real experimental data. One way to overcome this limitation is to use a secondary in-plane deformation approach with sophisticated models for which the anti-plane shear deformation isn't possible (Pucci & Saccomandi, 2013).

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In linear elasticity, the Linear Fracture Mechanics (LEFM) based on the superposition principle gives a three-dimensional analytic elastostatic field. A criticism and a review of the LEFM can be founded in the book of Bui (2007) in which some three-dimensional linear elastic problems were summarized. A recent review and generalization were done by Yosibash (2011). The question of the order of singularity near front was treated in Chaudhuri and Xie (2000) and Leblond and Torlai (1992) by different analytic and analytic/numerical methods. For solids with multiple cracks a review can be found in Kachanov (1993).

For nonlinear power-type constitutive laws behaviour with small deformation, the pioneer works of Hutchinson (1968); Rice (1966, 1967); Rice and Rosengren (1968) for a crack with traction-free surfaces under pure in-plane or pure out-of-plane shear loading conditions, showed that the asymptotic development is made by the same power-type singularities. Whereas the majority of analytic analysis inspired by the above papers are restricted to a pure mode I, II, III or mixed mode I/II, experimental (Rosakis & Ravi-Chandar, 1986) and numerical (Nakamura & Parks, 1990) investigations have confirmed that the three-dimensional cracked solid undergoes failure under mixed mode I/III or II/III. Nevertheless, there are few papers, based on two approaches, dealing with the determination of the analytic stress and deformation fields near the three-dimensional crack front under combined multi-axial mode. The first approach is based on the heuristic and phenomenological concept firstly introduced by Guo (1993a,b) who enriches the stress field by in-plane and out-of-plane constraints functions due to the stress tri-axiallity. The second approach uses the asymptotic development method with the perturbation technique to analyze three-dimensional crack under combined anti-plane and in-plane deformation by assuming an infinite long cylinder. It was shown that the singular exponent of the anti-plane deformation differs from the one of the in-plane deformation except for the linear hardening behaviour (Pan, 1990). An interesting review in the topic is made in the book of Yuan (2013).

Within the framework of finite deformation (Ogden, 1997), in the paste decades, only few works have been focused on the analysis of the fully three-dimensional deformation and stress fields. This is due to the formidable complexity of the mathematical problem (Ogden, 1997), in contrast to plane problem, which makes the boundary-value problem equations highly nonlinear and very difficult to solve analytically or even numerically. A generalisation from a plane to a pseudo-plane deformation problem with uniform axial extension Carroll and Rajagopal (1986); Hill and Shield (1986); Rajagopal and Wineman (1984) and non-uniform axial extension (Saccomandi, 2005) was conducted. Partial and exact solutions to some three-dimensional problems were done in a series of papers of Hill and his co-authors by exploiting the 'reciprocal equilibrium equations' for particular hyperelastic potentials Hill (1973, 2001); Hill and Lee (1989). Coupling between anti-plane and plane deformations fields in the boundary value problem was shown to exist for nonlinear hyperelastic potential which makes it hard to resolve and the uncoupled governing equations hold only for the linear Neo-Hookean material (Horgan & Saccomandi, 2003). Accordingly, Pucci and Saccomandi (2013) used a perturbed hyperelastic potential approach to deduce a secondary deformation due to a principal one. Elastostatic fields near the crack front of a hyperelastic solid was first analyzed by Knowles and Sternberg (1973, 1974) for plane deformation, Knowles and Sternberg (1983) for plane stress and Knowles (1977) for anti-plane deformation hypothesis. Among other researchers, the work of Stephenson (1982) is to be credited to have clarified the local structure characteristic nature of the elastostatic fields near the crack tip of a generalized Mooney-Rivlin solid under plane deformation kinematic condition and mixed boundary conditions at infinity (Mode I and II). It was shown that the crack opens symmetrically, under Mode II conditions, contrary to the predictions of linear theory. In other words, the nonlinear global crack problem cannot admit an antisymmetric solution. A review of this topic is presented by Long and Hui (2015) and some other comments are done (Mansouri, Arfaoui, Trifa, Hassis, & Renard, 2016). For anti-plane deformation kinematic condition, some necessary and sufficient mathematical conditions, restricted the hyperelastic potential form, are given by Knowles (1976) and Knowles (1977) for incompressible materials to admit non-trivial states of anti-plane shear (Karoui, Arfaoui, Trifa, & Hassis, 2015).

The analysis proposed in the present work is a first approach to examine the three-dimensional character of the singular elastostatic fields near the crack front of a long Neo-Hookean hyperelastic solid induced by an anti-plane shear transformation superposed to a plane transformation. The local (near the crack front) boundary value problem is formulated in a fully nonlinear Lagrangian framework which can be seen as the composition of two local boundary value problems: the plane transformation problem and the anti-plane problem (Grisvard, 2011). An asymptotic analysis is carried out in order to calculate the deformation and stress fields near the crack faces. The structure of the singular deformation field is examined in detail. Emphasis is placed on describing the crack- profile after deformation. Finally, it is to be mentioned that the resulted stress field is not the sum of stress field deduced from the two local boundary value problems. This is due to the nature of the nonlinear problem which induces stress field coupling between the in-plane and out-of-plane transformations.

2. Formulation of the global crack problem

Consider an isotropic homogeneous incompressible hyperelastic body $\mathcal{B}$ which, in its undeformed configuration, occupies an infinite region $\mathcal{R}_0$ fig. 1

$$\mathcal{R}_0 = \{\underline{x} | (x_1, x_2) \in \Omega_0, \ -\infty < x_3 < +\infty\}, \tag{1}$$

where $\underline{x}$ is the position vector of the particle in the undeformed configuration and Ω_0 denotes a cross-section of $\mathcal{R}_0$. Then, the plane domain Ω_0 can be described by a polar coordinates system

$$\Omega_0 = \{(r, \theta) | r \in [0, +\infty[, \ \theta \in [-\pi, \pi]\}. \tag{2}$$