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On buckling and post-buckling behavior of functionally graded micro-beams in thermal environment

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ABSTRACT

This study provides an exact solution for the size dependent buckling and post-buckling behavior of functionally graded (FG) micro-beams with arbitrary boundary conditions which are subjected to combined thermo-mechanical loading. To this end, a theoretical formulation including the effects of size dependency, elastic foundation and uniform temperature distribution is first derived using the modified couple stress theory and through the principle of minimum total potential energy. Next, the nonlinear equations governing bending and stretching behavior of FG micro-beams are uncoupled to a fourth-order ordinary differential equation. Finally, the differential operator method is utilized to exactly solve the decoupled equation. Also, a Fourier series solution is presented for doubly-simply supported FG micro-beams to show the importance of exact solution. In the numerical results section, the effects of the geometric ratios, material distribution, temperature variation, and material length scale parameter on the post-buckling behavior are discussed in detail. Findings show that the Fourier series solution is not able to correctly predict the post-buckling behavior of FG micro-beams, since the effect of flexural-extensional coupling stiffness term appearing in the natural boundary condition is ignored. Also, it is seen that critical values of axial traction obtained from the post-buckling analysis are significantly varied with the transverse force per unit length and temperature variation, while the buckling analysis predicts that buckling values of beam remain constant. Therefore, it can be concluded that the buckling analysis is inadequate for analyzing FG micro-beams under thermo-mechanical loadings.

1. Background

1.1. The size effect

Recently, the rise of small-scale manufacturing technologies and development of materials science have led to produce beams whose characteristic sizes are in the orders of micron and sub-micron. Micro-beams are widely used in micro- and nanoelectromechanical systems (MEMS and NEMS) e.g. atomic force microscopes, sensors, chemical sensing device, signal filtering, fluid and mass transport, actuators, and pumps. According to some experiments reported by researchers (Lam, Yang, Chong, Wang, & Tong, 2003; McFarland & Colton, 2005), the size effect plays a major role on the design parameters namely: the maximum stress, natural frequency and buckling load. The size dependent mechanical behavior of micro-structures is different from what predicted by the classical continuum theory due to lack of the length scale parameters in its consti-

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tutive relation. Hence, many researchers have tried to develop various theories for capturing the size effect by considering the material length scale parameters in their constitutive relations. In this regard, several non-classical continuum theories such as the couple stress (Toupin, 1962), nonlocal (Eringen, 1968), modified strain gradient (Lam et al., 2003) and modified couple stress (Yang, Chong, Lam, & Tong, 2002) theories have been presented until now. Toupin (1962) developed the couple stress theory in which the presence of the couple stress tensor with two higher order material length scale parameters is taken into account in addition to the classical force stress tensor. Considering symmetry of the couple stress tensor, Yang et al. (2002) proposed the modified couple stress theory whose length scale parameters are decreased to one parameter. In recent years, non-classical continuum theories have been broadly utilized to formulate and study the size-dependent mechanical behavior of micro-and nano-structures. Some examples of such works are found in Refs. Farokhi, Ghayesh, and Hussain (2016), Farokhi, Ghayesh, Gholipour, and Tavallaeinejad (2017), Ghayesh (2018), Rajasekaran and Khaniki (2017), Dehrouyeh-Semnani and Bahrami (2016), Taati, Nikfar, and Ahmadian (2012), Taati, Molaei, and Basirat (2014) and Taati, Molaei, & Reddy (2014).

1.2. Functionally graded materials

In the past three decades, a new class of materials known as "functionally graded materials" (FGMs) has emerged, in which properties are varied as a continuous function of position along certain direction(s) from one point to another. The disadvantage of interfaces in laminated composites can be eliminated by grading properties in a continuous manner. These materials are microscopically inhomogeneous and typically consist of ceramic and metal isotropic components. FGMs are primarily applied in structures subjected to prominent temperature gradients. Currently, the numerous demands have been raised for reaching multilayer MEMS with variable properties which are used in thermal environment. For example, the processes were defined to make an FG layer in micron size with the desired electrical and mechanical properties at its bottom and top surfaces (Witvrouw & Mehta, 2005). Hence, many researchers have been focused on the study of FG microstructures (please see: Asghari & Taati, 2013; Ghayesh, Farokhi, Gholipour, & Tavallaeinejad, 2018; Molaei, Ahmadian, & Taati, 2014; Molaei, Taati, & Basirat, 2014; Srividhya, Raghu, Rajagopal, & Reddy, 2018; Shenas, Ziaee, & Malekzadeh, 2017; Taati, 2016; Taati & Sina, 2018).

1.3. A literature review

Here, the some studies done on the buckling and postbuckling behavior of micro-structure are reviewed. For instance, Wang, Zhang, Ramesh, and Kitipornchai (2006) presented the elastic buckling analysis of micro- and nano-rods/tubes based on Eringen's nonlocal elasticity theory and the Timoshenko beam theory. Explicit expressions of critical buckling loads were obtained for axially loaded rods/tubes with various end conditions. Akgöz and Civalek (2014) investigated the thermo-mechanical buckling behavior of embedded FG micro-beams using the sinusoidal shear deformation beam and modified couple stress theories. The interaction between the FG micro-beam and the elastic medium was simulated by use of the Winkler elastic foundation model and the Navier solution was presented for doubly-simply supported FG micro-beams. They found that the effect of temperature change on the buckling characteristics become more distinguished for larger values of length-to-thickness (slenderness) ratio. Mohammadabadi, Daneshmehr, and Homayounfard (2015) studied the thermal effect on the buckling load of micro composite laminated beams with three kinds of supports including: hinged–hinged, clamped–hinged and clamped–clamped boundary conditions via the modified couple stress theory. Within the Euler–Bernoulli, Timoshenko and Reddy beam models, the governing equations were obtained and solved using the generalized differential quadrature method. Li and Hu (2015) presented a size-dependent nonlinear Euler–Bernoulli beam model to analyze the buckling and postbuckling behavior based on the non-local strain gradient theory. The stretching effect of the mid-plane was considered in this model and the postbuckling deflection and buckling values of simply supported beams were obtained using the Fourier series solution. Liang, Yang, Hu, and Shen (2016) provided buckling and vibration analyses of flexoelectric nano-films subjected to mechanical loads using the stain gradient theory and the classical Kirchhoff plate model. They comprehensively studied the flexoelectric effect on the buckling load and free frequencies and concluded that the critical load is not only influenced by the thickness of piezoelectric nanofilms, but also by the in-plane aspect ratio. Ansari, Shojaei, Gholami, Mohammadi, and Darabi (2013) investigated the thermal post-buckling characteristics of FG micro-beams undergoing thermal loads via the modified strain gradient theory. Based on the non-linear Timoshenko beam theory and the principle of virtual work, the static stability equations and corresponding boundary conditions were first derived and next solved through the generalized differential quadrature method in conjunction with a direct approach without linearization. Mohammadi and Mahzoon (2013) examined the thermal effect on the post-buckling behavior of micro-beams based on the modified strain gradient theory. Two general cases of beams with immovable (axial) and movable ends and different boundary conditions including hinged-hinged and clamped-clamped were studied using the Fourier series solution.

1.4. Present study

From the literature review, it is evident that the post-buckling analysis of micro-beams is limited to studies accomplished using the generalized differential quadrature methods (Ansari et al., 2013; Mohammadabadi et al., 2015; Wang et al., 2006) and analytical solutions (Li & Hu, 2015; Mohammadi & Mahzoon, 2013), which are based on the Fourier series expansions.