International Journal of Engineering Science 128 (2018) 127–150

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Nonlinear static and stability analysis of composite beams by the variational asymptotic method

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ARTICLE INFO

Article history:
Received 3 January 2018
Accepted 13 March 2018

Keywords:
Variational asymptotic method
Composite beams
Nonlinear static analysis
Elastic stability

ABSTRACT

The Variational Asymptotic Method (VAM) that is a powerful method for solving thin-walled beam problems is used for nonlinear static and stability analysis of composite beams. The finite difference and the finite elements methods are used for calculating elastic deformation of beams with clamped-free boundary conditions. The case of simply supported beams is also solved using linear and nonlinear formulations and the results are compared with each other. Next, the shooting method is used for performing nonlinear elasto-static analysis of cantilever beams. Finally by applying perturbations on the equilibrium equations and using an eigenvalue analysis, elastic stability of clamped-free composite beams is analyzed. The obtained results are compared with the available data in the literature that have been obtained using deformation analysis.

1. Introduction

There are many essentially one-dimensional (1-D) structural members (e.g. helicopter rotor blades) that are laterally flexible. Such members usually operate in the nonlinear range. One of the most powerful tools for analyzing such nonlinear problems is the Variational Asymptotic Method (VAM). This method was first introduced in Berdichevsky (1981) and can solve problems that have an inherently small dimension (e.g. beams, plates and shells). Interestingly, in VAM there are no restrictions on the geometry of the cross-section or on the materials of the structures for which the method can be applied. This method has been comprehensively discussed in Hodges (2006) and a brief review of it can be found in Ghorashi (2016).

The VAM method splits the 3-D geometrically nonlinear elastic analysis of composite beams into two parts. The first part is a (usually) linear 2-D analysis to determine the cross-sectional stiffness and mass matrices as well as the warping functions. The second part is a geometrically nonlinear 1-D beam model which utilizes the calculated stiffness and mass matrices in order to solve the nonlinear intrinsic equations of a beam.

Most of the research on VAM has been focused on the linear 2-D analysis in order to determine the cross-sectional stiffness and mass matrices. A few examples are Palacios and Cesnik (2005), Yu, Volovoi, Hodges, and Hong (2002), Yu, Hodges, and Ho (2012), and Yu and Hoges (2004). The 2-D analysis is performed by a computer program named VABS that is reviewed in Ghorashi and Nitzsche (2007).

There are also a few references that discuss the method of solution of the 1-D intrinsic equations of a beam. The ones that are frequently referred to in this paper are Ghorashi and Nitzsche (2008), Hodges (2006) as well as Ghorashi (2009, 2016). The present paper mostly expands the linear and nonlinear elasto-static analysis of composite beams given in Ghorashi (2009, 2016). Among other things, some of the results of the method developed in Ghorashi (2009, 2016) are

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https://doi.org/10.1016/j.ijengsci.2018.03.011
0020-7225/

Nomenclature

A	cross-sectional area of the undeformed beam in the x_2–x_3 plane
C	finite rotation tensor
E_i	moduli of elasticity ($i = 1,2,3$)
e_1	$[1 \quad 0 \quad 0]^T$
e_{ijk}	permutation symbol
F_i	elements of the column matrix of internal forces ($i = 1,2,3$)
f	applied forces vector per unit length
G_{ij}	shear moduli ($i, j = 1, 2, 3$)
K	deformed beam curvature vector$= k + \kappa$
k	undeformed beam curvature vector
L	length of the beam
M_i	elements of the column matrix of internal moments ($i = 1, 2, 3$)
m	applied moments vector per unit length
N	number of nodes
S	cross-sectional stiffness matrix
u_i	displacement field in the undeformed reference frame ($i = 1, 2, 3$)
x_i	coordinates system ($i = 1, 2, 3$)
x_1	longitudinal coordinate along the beam
x_2, x_3	cross-sectional coordinates
γ	$[\gamma_{11} \quad 2\gamma_{12} \quad 2\gamma_{13}]^T$
Δ	3×3 identity matrix
θ	Rodrigues parameters$= [\theta_1 \quad \theta_2 \quad \theta_3]^T$
κ_1	elastic twist
κ_i	elastic bending curvatures ($i = 2, 3$)
ν_{ij}	Poisson's ratios ($i,j = 1, 2, 3$)
$(\bar{\bullet})$	equilibrium values
$(\breve{\bullet})$	perturbations in space
$(\bullet)'$	$\frac{\partial(\bullet)}{\partial x_1}$
$(\tilde{\bullet})_{ij}$	$-e_{ijk}(\bullet)_k$

compared with those of the finite elements method (FEM). Also, in this paper, the finite difference method developed in Ghorashi (2009, 2016) is extended to analyzing the nonlinear elastic deformation of simply supported beams. The obtained results are compared with those of a linear static solution. The solution implements the methods developed in Hodges (2006) and Ghorashi (2009, 2012, and 2016) for transforming the governing nonlinear differential equations into the corresponding set of difference equations. These equations are then solved by implementing the boundary conditions.

Finally, in Hodges (2006), an elasto-static stability solution for composite beams using VAM has been given. Also elasto-dynamic stability analyses have been presented in Shang and Hodges (1995) and Cesnik, Shin, and Wilbur (2001). Such solutions are performed in two steps. First the steady state response is calculated. Then, the perturbed motion of the beam about the obtained steady state position is calculated by solving the perturbed steady state equations for small perturbations of the equilibrium variables. The present paper uses the shooting method for calculating the static equilibrium of cantilever beams made of composite materials. This is comparable to the calculation of steady-state solution of rotating beams given in Ghorashi (2009, 2012, and 2016). The obtained static solution is then perturbed and the resulting linearized problem is solved for its eigenvalues in order to perform the stability analysis. It is shown that buckling and elastic instability of composite beams can be well explained by the eigenvalues of the perturbation equation matrix.

2. Review of the cross-sectional modeling and 1-D formulation

Considering a beam model whose undeformed and deformed coordinate systems are shown in Fig. 1 and by using the cross-sectional stiffness matrix the constitutive law of the beam model can be expressed as, Hodges (2006),

$$\begin{Bmatrix} F \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B^T & D \end{bmatrix} \begin{Bmatrix} \gamma \\ \kappa \end{Bmatrix} \tag{1}$$

By inverting Eq. (1) and using the flexibility matrix one obtains the following alternative form,

$$\begin{Bmatrix} \gamma \\ \kappa \end{Bmatrix} = \begin{bmatrix} R & Z \\ Z^T & T \end{bmatrix} \begin{Bmatrix} F \\ M \end{Bmatrix} \tag{2}$$