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On vibrations of functionally graded viscoelastic nanobeams with surface effects

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ABSTRACT

This paper addresses the size-dependent free vibration of functionally graded viscoelastic (FGV) nanobeams including the simultaneous effects of the microstructure rotation and surface energy for the first time. Employing the Bernoulli–Euler beam theory, an internal damping mechanism based on Kelvin–Voigt model is adopted to simulate the viscoelastic behavior of the material. The modified couple stress theory and Gurtin–Murdoch surface elasticity theory are reconsidered and harnessed to capture the viscoelastic microstructure rotation and viscoelastic surface energy effects, respectively. The local-Cauchy stress, couple stress and surface stress tensors are obtained incorporating measures for the elastic and the viscous behaviors of the nanobeam. The elastic and viscous material properties of the bulk and surface of the FGV nanobeam are assumed to vary continuously in thickness direction according to a power law. A variational approach on the basis of D'Alembert's principle is employed to derive exactly the size-dependent governing differential equation and the associated nonclassical boundary conditions. An analytical expression is derived for the complex natural frequencies of a simply supported FGV nanobeam. In the context of linear viscoelasticity, the influences of different parameters such as the material damping, gradient index, material length-scale parameter, surface elasticity, surface residual stress, surface mass density, Poisson effect, thickness, and slenderness ratio on the free vibration of simply supported FGV nanobeams are comprehensively discussed. The results highlighted the profound effects of the small size, surface energy and viscoelastic behavior on the free vibration response of FGV nanobeams.

1. Introduction

The dramatic growth of nanotechnology has positively impacted the construction of micro- and nanoscales structural elements, which have been widely used in micro- and nanoelectromechanical systems (MEMS and NEMS) applications due to their extraordinary mechanical, electrical and thermal properties (Craighead, 2000; Ekinci & Roukes, 2005; Li, Bhushan, Takashima, Baek, & Kim, 2003; Sandberg, Svendsen, Molhave, & Boisen, 2005; Witvrouw & Mehta, 2005; Roy & Gao, 2009).

Functionally graded materials (FGMs) are advanced class of composite materials that made of the mixture of two materials, for instance metal and ceramic or polymer. In FGMs, material properties such as elasticity modulus, Poisson's ratio, shear modulus, mass density and thermal conductivity vary smoothly and continuously from one surface to another in the

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desired direction. Consequently, FGMs possess many advantages over the conventional layered composite materials, such as designability, smaller stress concentration, enhanced thermal properties, higher fracture toughness and reduction of thermal and residual stresses (Birman & Byrd, 2007; Udupa, Rao, & Gangadharan, 2014). At the same time, with the rapid emergence of nanotechnology, FG micro and nanostructures have wide applications in engineering fields such as MEMS and NEMS (Lee et al., 2006; Witvrouw & Mehta, 2005). In addition, the size effects were experimentally observed in mechanical properties of micro- and nanoscale structures (Chong & Lam, 1999; Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003; Ma & Clarke, 1995; Nix, 1989; Stölken & Evans, 1998). It is however crystal clear that the classical continuum mechanics theories failed to accurately predict the responses of such structures. Therefore, to overcome this shortcoming of the classical continuum mechanics, two different approaches have been proposed to capture these size effects; molecular dynamics (MD) simulations and nonclassical continuum mechanics. Comparing to the former, the later approach is more popular and has been widely used to investigate the mechanical behavior of elastic micro- and nanostructures because of its simplicity and computational efficiency (Thai, Thai, Vo, & Patel, 2017; Yue, Ru, & Xu, 2017).

In the literature, several size-dependent nonclassical continuum theories have been proposed, such as the nonlocal continuum field theory of Eringen (Eringen, 1972, 2002), couple stress theory (Koiter, 1964; Mindlin & Tiersten, 1962; Toupin, 1962), the strain gradient theory (Lam et al., 2003), the modified couple stress theory (Yang, Chong, Lam, & Tong, 2002) and surface elasticity theory (Gurtin & Murdoch, 1975, 1978). The most important advantage of the modified couple stress theory (MCST) over the aforementioned size-dependent theories, especially the couple stress theory (CST) which includes two higher order material length-scale parameters in addition to the two classical constants for isotropic elastic materials, the couple stress tensor is symmetric and involving only one additional higher-order material length-scale parameter besides two classical material constants. As a consequent, the relations of the CST are simplified and so far the MCST has attracted many researchers to correlate micro/nanoscale structures. In the context of MCST, there is a vast amount of literature on investigating the size-dependent static and dynamic mechanical behaviors of functionally graded elastic (FGE) *micro/nanobeams*; i.e. Akgöz and Civalek (2013), Al-Basyouni, Tounsi, and Mahmoud (2015) Arbind and Reddy (2013), Asghari, Rahaeifard, Kahrobaiyan, and Ahmadian (2011), Dehrouyeh-Semnani, Mostafaei, and Nikkhah-Bahrami (2016), Farokhi, Ghayesh, and Gholipour (2017), Ghadiri and Shafiei (2016), Ghayesh, Farokhi, and Gholipour (2017), Ghayesh, Farokhi, Gholipour, and Tavallaeinejad (2017), Ke and Wang (2011), Nateghi, Salamat-talab, Rezapour, and Daneshian (2012), Reddy (2011), Shafiei and Kazemi (2017), Shafiei, Mousavi, and Ghadiri (2016), Şimşek and Reddy (2013), Şimşek, Kocatürk, and Akbaş (2013), Thai, Vo, Nguyen, and Lee (2015) and Trinh, Nguyen, Vo, and Nguyen (2016). From these studies, it was found that incorporating the small scale effect via MCST shows a stiffness enhancement effect (stiffness–hardening) for FGE micro/nanobeams and thus changes their linear and nonlinear motion characteristics.

In the context of the continuum mechanics of micro/nanoscale structures, it has been proved that the saved energy in surface layers becomes comparable to that in the bulk because of the relatively high surface area to volume ratio, Lee and Rudd (2007). Thus, the crucial effects of surface tension and surface elasticity on the mechanical behavior of such elastic micro/nanoscale structures become considerable and cannot be neglected. In an effort to study surface elasticity of small-scale elastic materials, Gurtin and Murdoch (GM) (Gurtin & Murdoch, 1975, 1978) developed a theoretical framework of surface elasticity, and the related surface elasticity parameters were estimated, among others, by Miller and Shenoy (2000) using atomistic simulation. In GM theory, surface is considered as a mathematical zero thickness deformable membrane, fully adhered to the underlying bulk material and the constitutive equations of the bulk are the same as those in the classical theory of elasticity. In the past decade, the GM theory has been widely applied to study the effect of surface elasticity on the static and dynamic responses of *nanobeams* (Ansari & Sahmani, 2011; Ansari, Mohammadi, Shojaei, Gholami, & Rouhi, 2014; Lachut & Sader, 2012; Liu & Rajapakse, 2010; Wang, Zeng, & Wang, 2017; Zhao, Liu, & Wang, 2016), *nanoplates* (Eremeyev, Altenbach, & Morozov, 2009; Sapsathiarn & Rajapakse, 2017; Yue, Ru, & Xu 2017; Wang & Wang, 2012, 2013) and *nanofilms* (Lim & He, 2004; Zeng, Wang, Wang, & Guo, 2017). The influence of surface energy on the size-dependent bending, buckling and vibration responses of FGE *micro/nanobeams* has been investigated by few researchers; i.e. Ansari, Pourashraf, Gholami, and Sahmani (2017), Attia and Mahmoud (2015), Chen, Sun, and Li (2017), Hosseini-Hashemi and Nazemnezhad (2013), Saffari, Hashemian, and Toghraie (2017), Sahmani, Aghdam, and Bahrami (2015) and Sharabiani andYazdi (2013). These studies showed that the surface energy plays an important role in the mechanical behavior of FGE nanoscale beams.

Since surface elasticity theory characterizes the surface property and the modified couple stress theory describes the effect of material length-scale in the bulk, it is natural to combine both of them in investigating the mechanical behaviors of micro/nanoscale structures. In this regard, some models have been developed to investigate the simultaneous effects of material length-scale and surface energy on mechanical behavior of homogeneous *nanobeams* (Attia & Mahmoud, 2016; Sedighi & Bozorgmehri, 2016; Gao, 2015; Gao & Mahmoud, 2014; Shaat & Mohamed, 2014; Sourki & Hosseini, 2017; Zhang, Wang, Zhou, & Xue, 2016) and *nanoplates* (Shaat, Mahmoud, Gao, & Faheem, 2014; Wang, Wang, & Zhang, 2017; Gao, Zhang; 2016; Wang, Kitamura, & Wang, 2015; Zhang, Gao, & Tang, 2017). The number of studies in the literature on the combined effects of surface energy and the material length-scale on the mechanical behavior of FGE *micro/nanobeams* is still limited. Recently, Attia (2017a) developed an integrated nonlocal-couple stress-surface continuum elasticity model to investigate the bending, buckling and vibration responses of FGE nanobeams in the framework of the differential form of Eringen's nonlocal continuum model (DENCM). This model was extended by Shanab, Attia, and Mohamed (2017) to study the size-dependent nonlinear bending response of FGE nanobeams with different boundary conditions, including the combined effects of couple stress and surface energy. Collections of results on the development of the higher-order continuum models for capturing size