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A combined accretion and surface growth model in the framework of irreversible thermodynamics

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ABSTRACT

We develop in this contribution models for material accretion and surface growth in the framework of the thermodynamics of irreversible phenomena. Accretion is a general situation in physics, encompassing phenomena like masonry, gravitational accretion, chemical vapor deposition, or volcanic and sedimentary rock formation. Surface growth in a biological context results from the incremental accretion of new tissue onto the boundary of a solid body, due to the activity of generating cells which produce new tissue, this last process deserving the coinage surface growth. The classification into pure accretion occurring without growth and surface growth is concomitant to the definition of the growth velocity as the difference between the total velocity and the accretion velocity, this last quantity being defined as the velocity of the surface or interface occupied by the set of generating cells in a biological context. Moving (resp. fixed) generating cells or material points correspond to the respective situations of surface growth in a biological context and to accretion in physics. Based on this classification, we first analyze the situation of pure surface growth occurring in an elastic solid body, under the umbrella of the thermodynamics of irreversible phenomena. The situation of mass accretion of a spherical domain is given as an illustration. This framework is next enlarged to material accretion accompanied by surface growth, whereby an evolution law for the growth velocity gradient is obtained versus a conjugated driving force. Both problems of accretion and surface growth are coupled through the accretion velocity. Numerical simulations are performed in the biomechanical context of bone external remodeling to illustrate the general situation of combined accretion and surface growth.

1. Introduction

Surface growth classically refers to the accretion of initially free solid particles onto a surface; in physics, it usually means processes where material reorganize on the substrate onto which it is deposited (like epitaxial growth), but principally to phenomena associated to phase transition, whereby the evolution of the interface separating the phases produces a crystal (Kessler, 1990; Langer, 1980). Common examples of surface growth are e.g. the solidification of water close to the freezing temperature associated to the motion of the ice-water interface, chemical vapor deposition, volcanic and sedimentary rock

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formation, the build-up of ice and accumulation of snow (Brown, Evans, & LaChapelle, 1972), the formation of planetary objects in the context of gravitational accretion (Brown & Goodman, 1963; Ganghoffer, Rahouadj, Boisse, & Forest, 2016), equilibrium shape configurations for rubble piles, the growth of biological tissues (Skalak, Farrow, & Hoger, 1997). It also encompasses many technological processes and man-made constructions relying on the accumulation of material from an existing support, including buildings, metal solidification (Schwerdtfeger, Sato, & Tacke, 1998), electrolytic deposition, layer-by-layer gluing of composites, and all structural components built by additive manufacturing, which is a rapidly developing technique.

From a biological perspective, surface growth refers to mechanisms tied to accretion and deposition occurring mostly in hard tissues, and is active in the formation of teeth, seashells, horns, nails, or bones (Thompson, 1992). Many living or biological tissues are the result of a surface growth process, including trees, bones, soft tissues. A landmark in this field is (Skalak, Dasgupta, & Moss, 1982; Skalak, Farrow, & Hoger, 1997) who describe the growth or atrophy of part of a biological body by the accretion or resorption of biological tissue lying on the surface of the body. Surface growth of biological tissues is a widespread situation, with may be classified as either fixed growth surface (e.g. nails and horns) or moving growing surface (e.g. seashells, antlers). Models for the kinematics of surface growth have been developed in (Skalak et al., 1997), with a clear distinction between cases of fixed and moving growth surfaces, see (Ganghoffer, 2010a, 2010b; Garikipati, 2009) for a recent exhaustive literature review.

A clear distinction exists between bulk and surface growth, when considering their modeling assumptions in the context of continuum mechanics. Volumetric growth preserves material points, so that only mass density or specific volume change (this is an artifact to model new particles being squeezed between already existing ones), and the natural (stress-free) configuration of the body evolves with growth (Ben Amar & Ciarletta, 2010; Ben Amar & Goriely, 2005; Dervaux & Ben Amar, 2011). To the contrary, surface growth introduces new material points that are added or removed from the body boundary, so that the very notion of a reference configuration loses its meaning. In addition, the natural configuration of the body now depends on the history of the loading and deformation, in addition to the accretion flux and velocity. Theoretical works on the mechanics of surface growth trace back to the 1940s with the work of Southwell (1941) who analyzed the process of wire winding of thick-walled cylinders manufactured by wire winding of an initial elastic tube. In the early sixties, this was followed by a study by Brown and Goodman (1963) of the problem of a growing planet subjected to self-gravity. The authors solved the incremental elastic problem for a hollow linear elastic sphere subjected to a traction boundary condition originating from the infinitesimal weight of the added layer replacing the self-gravity load. This work was further extended by Kadish, Barber, and Washabaugh (2005) to a rotating solid sphere, making the decomposition of the stress tensor into a compatible time-dependent part and an incompatible time-independent part associated to the residual stresses. Most of the theoretical works on the mechanics of surface growth restrict to the linear framework. There is a wide literature on the mechanics of surface growth by Russian researchers, who mostly produced formal works, see (Manzhirov, 2017) and references therein. Let mention especially the work of Drozdov (1998a, 1998b), who introduced a multiplicative decomposition of the transformation gradient into elastic and plastic terms in finite plasticity, and solve many examples. More recently, Tomassetti, Cohen, and Abeyaratne (2016) developed a model for accretion whereby material is deposited on the inner surface of a solid (considering a model problem inspired by experiments on the mobility of actin), due to the diffusion of particles. Detachment of particles is supposed to occur at the same time at the outer surface, the balance of both phenomena corresponding to the so-called treadmilling regime. Accretion is considered by the authors as a non-equilibrium irreversible process, and the driving force determined from the evaluation of the dissipation rate involves the difference of chemical potential of a particle before and after accretion, the solid strain energy and the radial normal stress. Sozio and Yavari (2017) analyze the accretion of cylindrical and spherical nonlinear elastic solids in the geometric framework of Riemannian material manifolds.

A clear distinction is made in this contribution between accretion and surface growth, depending whether the aggregated material produces or not a new phase by internal mechanisms, as for instance for bone external remodeling in the context of biomechanics. More precisely, the following classification is adopted:

- Pure accretion without surface growth: this means that the material deposited onto the surface do not generate new material *per se*. From a kinematic viewpoint, it means that the growth velocity is nil, so that the total velocity is the sum of the accretion velocity and the elastic velocity due to the incompatible accretion process. In general, surface accretion leads to the development of both elastic surface and volumetric ‘strains’ leading to internal stresses; as a novel aspect in this contribution, the elastic surface strains shall be related to the (surface) stresses by a specific constitutive law specific to the interface separating the existing bulk material to the newly adhering material. Surface accretion without surface growth can be illustrated in masonry constructions which are built up in successive layers of bricks in the presence of gravity loads (Bacigalupo & Gambarotta, 2012). The stress and displacement fields in statically indeterminate structures subjected to gravity as the dominant load are significantly affected by the construction sequence and thus strongly differ from the corresponding ones obtained by applying gravity loads on the finished structure.
- Accretion with concomitant surface growth: this is the more general situation, which is nevertheless specific to living tissues, requiring considering both a model for the accretion velocity of the set of generating cells, and a model for the surface growth process caused by the generating cells. A typical illustrative situation in a biological context is bone external remodeling, which involves specialized cells (osteoblasts and osteoclasts) removing and producing the new mineral phase forming new bone.