International Journal of Engineering Science 127 (2018) 80–91

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

A refined Maxwell's scheme for calculating effective properties of composite layers

Ivan I. Argatov[a], Federico J. Sabina[b,*]

[a] Institut für Mechanik, Technische Universität Berlin, Berlin 10623, Germany
[b] Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas, Universidad Nacional Autónoma de México, Apartado Postal 20-126, Delegación, Alvaro Obregon CDMX 01000, México

ARTICLE INFO

Article history:
Received 4 October 2017
Accepted 8 December 2017

Keywords:
Effective properties
Composites
Asymptotic modeling

ABSTRACT

Effective properties of a composite coating layer bonded to an elastic substrate are evaluated by taking into account both the position of the representative volume element (RVE) within the coating layer and the substrate properties. The RVE characteristic size is explicitly introduced into the analysis as well. The size effects are incorporated by means of the two-layer Green's function. It is shown that the effective response of a composite coating in anti-plane deformation is determined by the thickness-variable shear modulus. A distinguishing feature of the developed refined Maxwell's scheme is that it allows for arbitrary anisotropy of the effective inclusion, which is assumed to be of circular shape.

1. Introduction

In recent years, there has been shown an increasing interest to develop more accurate evaluation schemes for the effective properties of composite materials (Rodin & Weng, 2014; Sevostianov & Kachanov, 2014). The majority of the theoretical studies concern the bulk properties of a composite regarded as a multi-phase material, and this widely adopted approach is quite accurate provided the correlation length of each phase is much less than the characteristic sizes of a given structural element made of the composite material. In their recent paper, Fleck and Willis (2016) have posed the question about the effective properties of a composite material with accounting for the geometry of structural element, in which the composite material has been implemented, thereby introducing into the analysis a characteristic length scale responsible for the size of the structural element.

In particular, in the case of a thin elastic composite coating bonded to a finite thickness elastic substrate, the Fleck–Willis approach aims to predict the elastic properties of the surface composite coating in anti-plane shear by taking into account not only the coating thickness but also the presence of substrate with differing elastic properties. Specifically, in their original stochastic approach, Fleck and Willis (2016) utilize the variational method of Hashin and Shtrikman (1963) to derive the Hashin–Shtrikman bounds (Willis, 1977) and self-consistent estimates (Hill, 1965) for both composite coatings and composite sandwich layers that are embedded between two substrates. In particular, for a surface coating of thickness h, the half-plane Green's function is exploited, as the results are presented in the limit $H/h \to \infty$, where H is the substrate thickness. It turns out that, within the Hashin–Shtrikman approximation, the response of the coating layer surprisingly does not depend upon the substrate properties.

* Corresponding author.
E-mail addresses: ivan.argatov@gmail.com (I.I. Argatov), fjs@mym.iimas.unam.mx (F.J. Sabina).

https://doi.org/10.1016/j.ijengsci.2017.12.005
0020-7225/

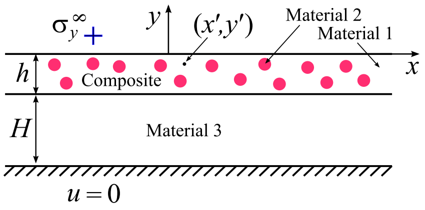

Fig. 1. A composite coating layer, comprising a two phase composite adhered to a an elastic substrate, under an imposed longitudinal shear traction σ_y^∞.

The study of Fleck and Willis (2016) revealed the existence of boundary layers of increased compliance at the free surface. It should be emphasized that though the boundary layer phenomenon has been known previously (Bakhvalov & Panasenko, 1989; Luciano & Willis, 2003), the new approach means that the effective response of a thin composite coating should be regarded as that of a certain thickness-inhomogeneous layer.

In the present work, we make use of Maxwell's homogenization methodology (McCartney, 2010; Torquato, 2002; Mityushev and Rylko, 2013) to develop a deterministic Fleck–Willis approach for calculating effective properties of composite coatings bonded to elastic substrates. With this aim, we construct a refined Maxwell's scheme that accounts for the position of the representative volume element (RVE) within the coating layer by employing the two-layer Green's function, thus explicitly introducing the elastic properties of the substrate into the homogenization scheme.

The paper is organized as follows. In Section 2, we develop a refined Maxwel's homogenization scheme for two-dimensional problems of anti-plane deformation of multi-phase composite with isotropic constituents. Section 3 illustrates the obtained results for two-phase isotropic particulate coatings, in particular, by comparison with the corresponding solutions given by Fleck and Willis (2016). In Sections 4 and 5, we outline the discussion and draw our conclusions.

2. Theory

2.1. Statement of the anti-plane shear problem for a composite layer bonded to a homogeneous elastic substrate

Following Fleck and Willis (2016), we consider the shear deformation of a composite layer of height h representing a random M-phase composite, bonded to a homogeneous substrate of height H made from $M+1$ material (see Fig. 1).

Let $u(x, y)$ be the only non-vanishing component of displacement field in the piecewise homogeneous and isotropic elastic body undergoing anti-plane deformation. Then, the only non-vanishing infinitesimal shear strain components are $e_x = u_{,x}$ and $e_y = u_{,y}$, where a comma before the suffixes x and y denotes partial differentiation with respect to the respective coordinate.

The corresponding shear stress components $\sigma_x \equiv \sigma_{zx}$ and $\sigma_y \equiv \sigma_{zy}$ are

$$\sigma_\alpha = \mu e_\alpha = \mu u_{,\alpha}, \tag{1}$$

where the shear modulus $\mu(\mathbf{x})$ takes a constant value μ_r when a given point $\mathbf{x} = (x, y)$ lies in material of type r. In what follows (as in Eq. (1)), it is assumed that a Greek suffix takes the values of x or y. Also, the Einstein convention is used to denote summation over a repeated Greek suffix.

In the absence of body forces, the equilibrium equation $\sigma_{\alpha,\alpha} = 0$ requires that $u_{,\alpha\alpha} = 0$ within each phase, i.e.,

$$-\mu_r u^{(r)}_{,\alpha\alpha} = 0, \quad \mathbf{x} \in \Omega_r, \quad r = 1, 2, \ldots, M+1, \tag{2}$$

where Ω_r is a union of individual domains ω_j^r, $j = 1, 2, \ldots, M_r$, occupied by material of phase r.

In the case of ideal contact between the phases, the following interface conditions are imposed inside the composite layer:

$$\mu_r \frac{\partial u^{(r)}}{\partial n} = \mu_1 \frac{\partial u^{(1)}}{\partial n}, \quad u^{(r)} = u^{(1)}, \quad \mathbf{x} \in \Omega_r, \quad r = 2, \ldots, M. \tag{3}$$

For the sake of concreteness, Eq. (3) assume that $M-1$ phases are embedded in a matrix Ω_1.

At the layer/substrate interface $y = -h$ (at the so-called bottom surface of the layer) the following conditions of perfect contact with the elastic substrate are imposed:

$$\mu_1 \frac{\partial u^{(1)}}{\partial n} = \mu_{M+1} \frac{\partial u^{(M+1)}}{\partial n}, \quad u^{(1)} = u^{(M+1)}, \quad y = -h. \tag{4}$$

In turn, the elastic substrate of thickness H is assumed to be bonded to a rigid substrate, so that

$$u^{(M+1)} = 0, \quad y = -(h+H). \tag{5}$$

On the other side, on the layer top surface, a longitudinal shear traction σ_y^∞ is supposed to be applied, i.e.,

$$\mu_1 \frac{\partial u^{(1)}}{\partial n} = \sigma_y^\infty, \quad y = 0. \tag{6}$$