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Efficient numerical solution of the hydraulic fracture problem for planar cracks

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ABSTRACT

An infinite isotropic elastic medium with a planar crack is considered. The crack is subjected to pressure of fluid injected inside the crack at a point of its surface. The fluid is viscous newtonian, incompressible, the medium is impermeable. Description of the crack growth is based on the lubrication equation (local balance of the injected fluid and the crack volume), the elasticity equation for crack opening caused by fluid pressure, Poiseulle equation related the fluid flux with crack opening and the pressure gradient, and the criterion of crack propagation of linear fracture mechanics. The crack growth is simulated by a series of discrete steps. Each step consists of three stages: increasing the crack volume by a constant crack size, crack jump to a new size defined by the fracture criterion, and filling the new crack configuration by the fluid presented in the crack. The problem is ill-posed and requires specific methods for numerical solution. The proposed method is based on an appropriate class of approximating functions for fluid pressure distributions on the crack surface and the theory of solution of ill-posed problems. Example of crack growth in a homogeneous and isotropic elastic medium is considered, influence of the fluid viscosity on the process of crack growth is studied.

1. Introduction

For importance in gas and oil extraction technology, the process of hydraulic fracture has been the object of intense theoretical and experimental studies for about sixty years. Various aspects of this process were discussed in a number of works. Most publications before 21-st century are mentioned in the books of Valko and Economides (1995) and Economides and Nolte (2000). Surveys of more recent publications can be found in Adachi, Siebrits, Peirce, and Desroches (2007), Detournay (2016), and Lecampion, Bunger, and Zhang (2018).

In this work, an infinite isotropic elastic medium with a planar crack is considered. The crack is subjected to pressure of fluid injected inside the crack at a point x^0 of its surface with positive injection rate $Q(t)$, t is time (Fig. 1). The fluid is viscous newtonian, the medium is impermeable. Description of the crack growth is based on local balance of the injected fluid and the crack volume, the elasticity equation for crack opening caused by fluid pressure, Poiseulle equation related the fluid flux with crack opening and the pressure gradient, and the criterion of crack propagation of linear fracture mechanics. It was shown (see, e.g., Adachi et al., 2007; Valko & Economides, 1995) that the problem is reduced to a non-linear differential

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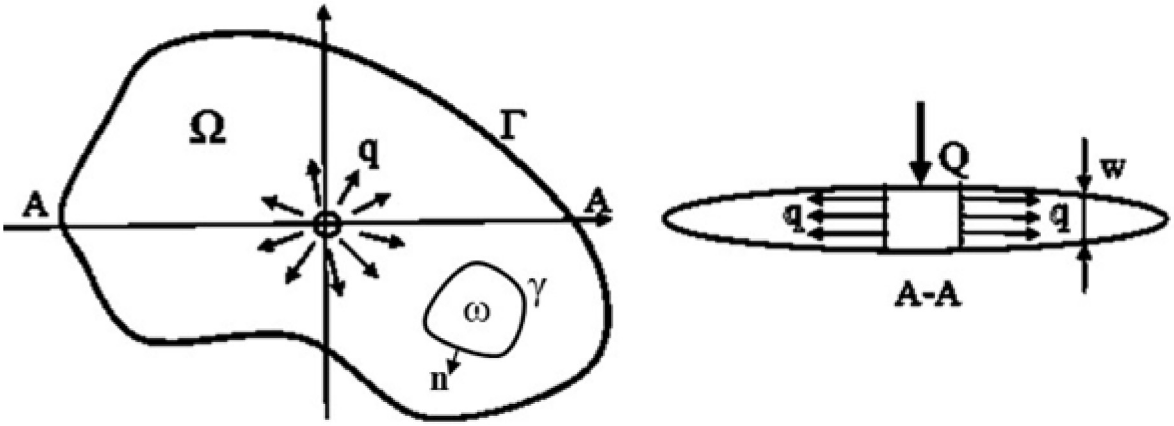

Fig. 1. A planar crack subjected to fluid injection.

equation with respect to crack opening $w(t,x)$ and fluid pressure $p(t, x)$ on the crack surface Ω (the lubrication equation)

$$\frac{\partial w(t,x)}{\partial t} = \mathrm{div}\left[\frac{\mathrm{w(t,x)}^3}{12\eta}\nabla \mathrm{p(t,x)}\right]. \tag{1}$$

Here x is a point on the crack surface, η is fluid viscosity. The pressure on the crack surface $p(t, x)$ and crack opening $w(t,x)$ relate by the equation (section 2)

$$p(t,x) = \frac{\mu}{4\pi(1-\nu)}\int_\Omega \frac{\Delta' w(t,x')}{|x-x'|}d\Omega', \ \ x \in \Omega. \tag{2}$$

Here μ and ν are shear modulus and Poisson ration of the medium, Δ is 2D-Laplace operator. It is assumed in the conventional theory of hydraulic fracture that in the process of crack propagation, the fracture criterion of linear fracture mechanics

$$K_I(t,x) = K_{Ic}(x), \quad x \in \Gamma(t) \tag{3}$$

is satisfied at all the points of crack contour $\Gamma(t)$. Here $K_I(t, x)$ is the stress intensity factor for fracture mode I at the crack contour, and $K_{Ic}(x)$ is fracture toughness of the medium. Eqs. (1)–(3) compose a closed system of equations for hydraulic fracture crack propagation in homogeneous isotropic elastic media with varying in space fracture toughness. Analytical solutions of this system do not exist even in the simplest cases, e.g., for a constant fracture toughness and a penny shape crack, and only numerical methods are efficient.

For numerical solution, Eq. (1) should be discretized with respect to time t and presented in the form

$$w(t+\Delta t,x) = w(t,x) + \mathrm{div}\left[\frac{\mathrm{w(t,x)}^3}{12\eta}\nabla \mathrm{p(t,x)}\right]\Delta \mathrm{t}. \tag{4}$$

Thus, if functions $w(t,x)$ and $p(t, x)$ are known at moment t, one can calculate crack opening $w(t+\Delta t,x)$ at moment $t+\Delta t$. The difficulty in constructing $w(t+\Delta t,x)$ is that new crack area $\Omega(t+\Delta t)$ is not known in advance. If we assume that in the time interval Δt, the crack contour does not change ($\Gamma(t+\Delta t) = \Gamma(t)$), Eqs. (4) and (2) determine crack opening $w^+(t+\Delta t,x)$ and pressure distribution $p^+(t+\Delta t,x)$ on the crack surface $\Omega(t)$ at the moment $t+\Delta t$. For a positive fluid injection rate Q, stress intensity factor $K_I^+(t+\Delta t,x)$ on the crack contour $\Gamma(t)$ can exceed $K_{Ic}(x)$. Now, we can change contour $\Gamma(t)$ in such a way that the fracture criterion is satisfied at the points of the new crack contour $\Gamma(t+\Delta t)$. But in order to do this we have to define the pressure distribution on the new crack surface $\Omega(t+\Delta t)$ since pressure $p^+(t+\Delta t,x)$ changes if the crack area changes. In any numerical algorithm, an assumption of the new pressure distribution on the crack surface $\Omega(t+\Delta t)$ should be accepted (explicitly or implicitly), but a unique natural definition of such a distribution does not exist.

The second principal difficulty in numerical solution of Eqs. (2)–(4) is that the right hand side of (4) depends on four spatial derivatives of functions $w(t,x)$. It is known that derivation is an ill-posed operation (Tikhonov & Arsenin, 1977), and small errors in calculation $w(t,x)$ result in large errors in calculation of $w(t+\Delta t,x)$. Since Eq. (4) should be solved many times (for a number of intervals Δt which the total time of injection is divided into), the errors accumulate and a reliable solution can be lost. The same difficulty appears if we use any conventional numerical method, e.g., finite element method or extended finite element method, for constructing pressure distribution for known crack opening $w(t,x)$: small errors in $w(t,x)$ result in large errors in the corresponding pressure distribution $p(t, x)$. In the most works on numerical solution of the hydraulic fracture problems, these two difficulties are not pronounced and discussed.

In the case of penny shape cracks, a way of overcoming these difficulties was proposed in Kanaun (2017). In this work, the hydraulic fracture process of crack growth is simulated by a series of discrete steps. Each step consists of three stages: