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Nonlinear mechanical behaviour of microshells

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ABSTRACT

This study examines the nonlinear large-amplitude static and dynamic responses of a doubly curved shallow microshell in the framework of the modified couple stress (MCS) theory. To this end, the expressions for the classical and higher-order stresses and strains are consistently derived in an orthogonal curvilinear coordinate system employing the Novozhilov shell theory. The strain energy of the system is then consistently derived utilising the Novozhilov shell formulations in the framework of the MCS theory. The kinetic energy of the microshell is obtained while accounting for all out-of-plane and in-plane displacements. Furthermore, the work of the distributed out-of-plane load is accounted for and the energy dissipation is taken into account via the Rayleigh energy dissipation function. An assumed-mode technique is utilised to expand the out-of-plane and in-plane displacements via series expansions. The Lagrange equations are then utilised to derive the discretised equations of motion in the form of a set of nonlinearly coupled ordinary differential equations (ODEs). This set of nonlinear ODEs is solved making use of a continuation technique (for the nonlinear static and dynamic analyses) as well as an eigenvalue extraction method (for the linear natural frequency analysis). Extensive numerical simulations are carried out for both static and dynamic cases and the effects of different parameters, such as the radius of curvature, the magnitude and direction of the applied distributed load, and the small-scale parameter are investigated. The numerical results are constructed in the form of nonlinear static deflection curves, nonlinear dynamic frequency-amplitude diagrams, time traces, and phase-plane portraits.

1. Introduction

Microelectromechanical systems (MEMS) are microdevices consisting of integrated mechanical and electrical components, the former commonly being a micro-scale beam, plate, or shell (Akgöz & Civalek, 2013, 2011; Asghari, Kahrobaiyan, & Ahmadian, 2010; Baghani, 2012; Dehrouyeh-Semnani, 2014; Dehrouyeh-Semnani, BehboodiJouybari, & Dehrouyeh, 2016; Farokhi, Ghayesh, & Gholipour, 2017b; Farokhi, Ghayesh, & Amabili, 2013a; Farokhi, Ghayesh, Gholipour, & Hussain, 2017a; Ghayesh & Farokhi, 2015, 2017; Ghayesh, Amabili, & Farokhi, 2013b; Ghayesh, Farokhi, & Amabili, 2013a; Ghayesh, Farokhi, Gholipour, Hussain, & Arjomandi, 2017a; Hosseini & Bahaadini, 2016; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Karparvarfard, Asghari, & Vatankhah, 2015; Kong, Zhou, Nie, & Wang, 2008; Mojahedi & Rahaeifard, 2016; Taati, 2016). MEMS devices can be utilised in numerous applications such as in microactuators, microresonators, micromirrors, microgyroscopes, and microsensors. In order to have a better understanding of the behaviour of these microdevices, one can study the

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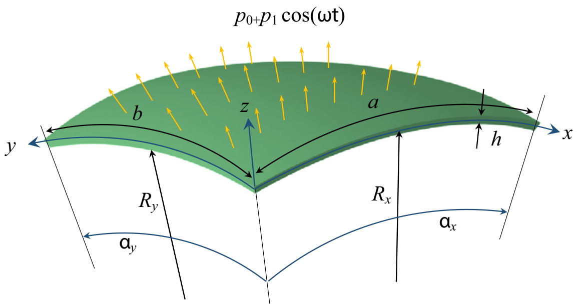

Fig. 1. Schematic of the doubly curved microshell in an orthogonal curvilinear coordinate, under static and dynamic loadings.

behaviour of its components. As mentioned, the mechanical components of MEMS devices are commonly microelements which can be modelled via beam, plate, or shell theories. The goal of this study is to examine the nonlinear static and dynamic behaviours of shallow microshells, which can be a representation of two-dimensional elements with initial curvature. Furthermore, micro-scale elements usually display size-dependent behaviour relating to their characteristic micro-scale size (Dai, Wang, & Wang, 2015; Farokhi & Ghayesh, 2015a, b; Farokhi, Ghayesh, & Amabili, 2013b; Ghayesh & Amabili, 2014; Ghayesh & Farokhi, 2015; Ghayesh, Farokhi, & Amabili, 2013c, 2014; Ghayesh, Farokhi, & Hussain, 2016; Gholipour, Farokhi, & Ghayesh, 2015; Li & Pan, 2015; Şimşek, 2010; Tang, Ni, Wang, Luo, & Wang, 2014). Such a size-dependent characteristic (Farokhi & Ghayesh, 2018; Farokhi & Ghayesh, 2016, 2017; Ghayesh, Farokhi, Gholipour, & Tavallaeinejad, 2018; Ghayesh, Farokhi, Gholipour, & Hussain, 2017e; Ghayesh, Amabili, & Farokhi, 2013d; Ghayesh, Farokhi, & Gholipour, 2017b, c; Ghayesh, Farokhi, Gholipour, & Tavallaeinejad, 2017d) can be modelled via employing higher-order continuum theories; the present study employs a modified version of the couple-stress strain gradient theory, also known as the modified couple stress (MCS) theory (Farokhi, Ghayesh, Gholipour, & Tavallaeinejad, 2017c; Lou, He, & Du, 2015). Microshells of double curvature are prone to snap-through buckling as well as modal interactions and internal resonances, which could be beneficial or detrimental depending on the application; the goal of this study is to develop an accurate model for the microshell and to examine the effect of radius of curvature on snap-through buckling, modal interactions, and small-scale effects.

Although there are many studies in the literature concerning the static/dynamic characteristics of microbeams and microplates, the number of studies on the behaviour of microshells is not large. For instance, a buckling analysis was performed by Lou, He, Wu, and Du (2016) on the simply-supported microshells made of functionally graded materials, while accounting for the small-size effects employing the MCS theory. The size-dependent equations of motion of a functionally graded cylindrical shell were derived by Tadi Beni, Mehralian, and Razavi (2015) employing the MCS theory; they discretised the equations and examined the free vibration characteristics the system.

The present study investigates, for the first time, the nonlinear static and dynamic characteristics of a doubly curved microshell while accounting for: (i) small-scale effects, (ii) geometric nonlinearities, and (iii) out-of-plane and in-plane displacements and inertia, while employing a high-dimensional discretised model. In particular, the higher-order strain and stress components are consistently derived in the framework of the MCS theory. Employing the Novozhilov doubly curved shell theory, the strain and kinetic energies of the system are derived. Taking into account the work of the external distributed load as well as the energy dissipation, the nonlinear discretised equations of motion are derived with the aid of Lagrange equations in conjunction with an assumed-mode method. A large number of modes are retained in the discretised model to ensure reliable predictions even at large-amplitude displacements. A well-optimised continuation technique is utilised to solve the high-dimensional nonlinearly coupled discretised model for both static and dynamic cases. The effects of various parameters are examined and the results are presented in the form of nonlinear static deflection curves, frequency-amplitude responses, time traces, and phase-plane portraits.

2. Model development and discretisation

The system under consideration is a doubly curved microshell of curvilinear dimension a, in the x direction, and curvilinear dimension b, in the y direction, and thickness h, in the z direction; the detailed schematic of the system is demonstrated in Fig. 1, describing the microshell in the coordinate system (x, y, z) which is an orthogonal curvilinear one; z denotes the normal to the shell surface, while the other coordinates, i.e. x and y, can be expressed in terms of the microshell principal radii of curvature (R_x and R_y) and angular coordinates (α_x and α_y) as $x = R_x\alpha_x$ and $y = R_y\alpha_y$.

In this section, the Novozhilov shell theory, in the framework of the MCS theory, is employed so as to derive the nonlinear equations of motion of a doubly curved microshell. In particular, the MCS theory formulations are consistently derived in an orthogonal curvilinear coordinate system; the Novozhilov doubly curved shell theory is then extended in the framework of the higher-order MCS theory to account for the small-scale effects.

Denoting the doubly curved microshell mid-plane displacements by $u(x,y,t)$ and $v(x,y,t)$, in the x and y directions, and by $w(x,y,t)$ in the z direction, the formulations for the displacements of an arbitrary point of the microshell, positioned at a