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Short Communication

On the impact of tangential traction on the crack surfaces induced by fluid in hydraulic fracture: Response to the letter of A.M. Linkov. Int. J. Eng. Sci. (2018) 127, XX–XX

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ABSTRACT

In response to the "critical comments" by Dr. Linkov concerning our publication Wrobel et al. (2017), we will demonstrate here the major faults in the logic of his arguments. We uphold the conclusions from Wrobel et al. (2017), in particular that the hydraulically induced shear stresses on the fracture faces may play an important role in the HF process and its numerical simulation, especially in the viscosity-dominated regime.

We respond to the critical remarks of Dr. Linkov regarding our paper (Wrobel, Mishuris, and Piccolroaz, 2017, "Energy release rate in hydraulic fracture: Can we neglect an impact of the hydraulically induced shear stress?"); note that similar statements have been made by him in a different paper (Linkov, 2017) which, for some reason, he neglected to mention in his communication. Yet another motivation for our response is that this topic has already attracted substantial attention in the field of hydrofracturing (HF) (see, for example, Shen & Zhao, 2017).

Prior to responding to Dr. Linkov's criticisms point-by-point, we note that, in his concluding remark

> "*…the impact of the shear stress in the elasticity equation can be confidently neglected when solving practical problems of hydraulic fracturing…*"

Dr. Linkov is addressing a question that is *not the same* as the one originally posed in the title of our paper: we discussed *all* effects caused by hydraulically-induced shear stress on the fracture surface. In other words, his objections are to a statement that we never made.

We repeat that the three main points, related to the effect of shear stress, that were addressed in our work (highlighted by bullet points there) are:

A. Elastic response of the solid material,
B. Asymptotic near-tip behaviour of the solution,
C. Fracture propagation criterion.

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Of these points, Dr. Linkov discusses only the first one (and only partly—concerning the effect of the shear stress on the boundary integral equation), ignoring the more important points B and C.

Our response is as follows.

A. The effect of hydraulically-induced shear stress *on the boundary elasticity equation* is indeed relatively small (this was already stated in our paper, see Figs. 7–10). Note however that, contrary to Dr. Linkov's statement, this effect may not always be negligible: it is about 2–3% for the crack velocity and the crack opening and near 8% for the pressure at the crack inlet (in the viscosity-dominated regime).

Although points B and C—the main ones of our work—have not attracted Dr. Linkov's attention, we use this opportunity to highlight the key related issues.

B. **The crack-tip asymptotics remains the same regardless of the propagation regime**. This fact—which contradicts the commonly held viewpoint—becomes clear from the following two considerations. First, allowing different asymptotics in the viscosity dominated regime contradicts the modified integral equation (22) from Wrobel et al. (2017). Second, taking the shear stress into account and assuming the usual asymptotics for this regime, one obtains an infinite energy release rate.
C. **The energy release rate (ERR) criterion no longer coincides with the Irwin fracture criterion (regardless of the values of K_{Ic}).**

Taking the presence of shear stress—and its implications for the tip stress-strain fields—into account, the form of the general ERR criterion needs to be re-examined: it can be shown to be different from the Irwin fracture criterion typically used in HF models (see Section 3.2 of our work). Its significance is sufficient that it is also mentioned in the title of the paper. The modified fracture criterion now takes the form (see Eqs. (40) and (42) of our work):

$$K_{Ic}^2 = K_I^2 + 4(1-\nu)K_I K_f, \tag{1}$$

where K_{Ic} is the material toughness, K_I is the mode I SIF and K_f denotes the newly introduced factor reflecting the effect of the above-mentioned fluid-induced shear stress. Importantly, K_f assumes a finite value when $K_{Ic} = 0$, while $K_f \to 0$ as $K_{Ic} \to \infty$. This change in the ERR criterion is particularly significant in the viscosity dominated regime.

We now return to point A and discuss, point-by-point, the logical fallacies made by Dr. Linkov in this regard.

(1) His analysis relies on the following representation of the tangential stress at crack faces:

$$\tau(x,t) = \frac{M}{2}\frac{v(x,t)}{w(x,t)}, \tag{2}$$

where $v(x,t)$ is the fluid velocity within the fracture. Note that our work accounts for the full form of the equation, whereas Dr. Linkov only takes, in his Eq. (1), its asymptotic representation near the point $x = l(t)$. He claims that the following explains our "mistake":

"Unfortunately, they have not derived Eq. (1), which provided us with the quantitative estimations. Not having this equation, they formally tended r to zero when considering the ratio $\tau/|p|$ in Eq. (16) of their paper. Clearly, the ratio tends to infinity, what leads to an illusion that the shear stress should be accounted for in the elasticity equation."

- We point to Eq. (16) of our paper that *does* make use of the (rather trivial) Eq. (2);
- Further, the value of the mentioned ratio τ/p is not necessarily small as is commonly assumed; see Fig. 1 where we plot the reciprocal quantity, p/τ. This figure also shows that, in the case of Non-Newtonian fluid (considered by Linkov, 2017), the value of the ratio τ/p—and hence its importance—increases.

(2) Dr. Linkov discusses the value of the following ratio in the modified elasticity equation:

$$R_\tau(x,t) = -\frac{k_1\tau(x,t)}{k_2 w'_x(x,t)} = -\frac{Mk_1}{2k_2 w'_x(x,t)}\frac{v(x,t)}{w(x,t)}. \tag{3}$$

He performs an asymptotic analysis of this ratio at the fracture front and utilizes values of the constants and parameters that he considers "feasible in HF". He aims to find the range over which the shear stress is the dominant term. He concludes:

"Then Eq. (5) implies that the input of the shear traction $\tau(r)$ reaches 1% of the input of the conventional term $-\partial w/\partial x$ only at the distance r from the tip less than $1.67\cdot10^{-8}$ m; it reaches the level of 10% at the distance of $1.67\cdot10^{-11}$ m. This shows that the input of the shear stress may reach ten percent only at the distance of fractures of atomic sizes. Surely, it is beyond practical applications of HF."

We point to the following flaws in his analysis:

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