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Revisiting bending theories of elastic gradient beams

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ABSTRACT

In this paper, we discuss a problem of Bernoulli–Euler beam models development in the frame of the strain gradient and distortion gradient elasticity theories. Contradiction between some known size-dependent gradient beam models and analytical and numerical three-dimensional solutions found for the beam bending problems is shown. In particular, it is shown that in semi-inverse analytical solutions for a beam pure bending problem, the inverse squared dependence of normalized bending stiffness on the beam thickness could arise only due to wrong definition of the resultant bending moment and improper formulation of the boundary conditions on the top and bottom surfaces of the beam. In the correct semi-inverse solutions, finite values of normalized bending stiffness for ultrathin gradient beams arise. The obtained results are also confirmed on the basis of 3D FE simulations realized for a pure and cantilever beam bending problems in the frame of simplified strain gradient elasticity theory. As a result, it is shown that the only correct approach for Bernoulli–Euler beam models development in gradient theories should ensures the fulfillment of boundary conditions on the top and bottom surfaces of the beam and corresponds to a model that assumed a uniaxial stress state of the beam. The consistent variational algorithm for the correct gradient beam models derivation is proposed.

1. Introduction

From the practical point of view, the development of size-dependent beam models is related with the need of the correct description of the known experimental results for the micro-sized beams which bending stiffness non-classically depends on its thickness (see Kakunai, Masaki, Kuroda, Iwata, & Nagata, 1985; Lam, Yang, Chong, Wang, & Tong, 2003; Liebold & Müller, 2016; Poncharal, 1999; Stan, Ciobanu, Parthangal, & Cook, 2007). Size-dependent models should provide a precise modelling of micromechanical devices, MEMS, NEMS and ensure a correct processing of experimental data obtained using modern micro and nanosized sensors, detectors, atomic force microscopy, etc.

Size-dependent beam, plate and shell models are developed over the past fifty years. The first ones were elaborated in the frame of micropolar (Cosserat) and couple stress theories by Ellis and Smith (1968); Eringen (1967); Green and Naghdi (1967); Green, Naghdi, and Wainwright (1965). However, in the present paper these types of theories, including modified couple stress theory (see Park & Gao, 2006, Yang, Chong, Lam, & Tong, 2002), modified gradient elasticity theory (see Lam et al., 2003) and other Cosserat-type theories (see Altenbach, Altenbach, & Eremeyev, 2010) and Mindlin III form gradient theories (see Mindlin, 1964; Polizzotto, 2016) will be out of consideration. We will be focused on the beam models developed in the frame of distortion gradient and strain gradient theories, followed from the Mindlin Forms I and II, respectively

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(see Mindlin, 1964; Polizzotto, 2015, 2016). In the theories of this type, the components of rotation vector, curvature tensor and couple stress tensor are assumed to be equal to zero.

The first size-dependent Bernoulli–Euler beam models in the frame of strain-gradient elasticity were developed by Papargyri-Beskou, Tsepoura, Polyzos, & Beskos (2003) and by Lazopoulos (2003). These authors used two basically different approaches which are discussed in the present paper. These approaches are extended now to the plates and shells gradient theories. Thus, the basic principles discussed in the following are related not only to the beams models, but also to the plates and shells models.

In Papargyri-Beskou et al. (2003) the simple one-dimensional statement of the strain-gradient elasticity theory (in the form of Vardoulakis and Sulem (1995) including surface effects, but it does not matter for the present subject) was used with only axial non-zero components of strain ε_{xx}, strain gradient $\varepsilon_{xx,\,x}$, Cauchy stress τ_{xx} and double stress μ_{xxx} tensors. Therefore, only one axial total stress component $\sigma_{xx} = \tau_{xx} - \mu_{xxx,x}$ was taken into account for the resultant forces and bending moment derivation. Thus, the uniaxial stress state was assumed for the beam model developed in Papargyri-Beskou et al. (2003). It was obtained the sixth order governing equilibrium equation and boundary conditions of the Bernoulli–Euler gradient beam model based on variational approach. Specifically, the governing equation was the following:

$$D_0 w^{IV} - l^2 D_0 w^{VI} = q, \tag{1}$$

where $D_0 = EI$ is classical beam bending stiffness, E is material Yung's modulus, I is a moment of inertia of a beam cross section, $w = w(x)$ is a transverse deflection, $q = q(x)$ is a transverse load, x-axis is the axis of the beam and l is material length scale parameter which is denoted as g, c or ξ by different authors.

The same to Papargyri-Beskou et al. (2003) approach was also attended earlier in Vardoulakis, Exadaktylos, and Kourkoulis (1998) but in less systematic manner. In Peddieson, Buchanan and McNitt, 2003 governing Eq. (1) was also obtained as the special case of Eringen's non-local model, but no variational or boundary value problem statement was given. The same uniaxial stress assumption was also used in Wang and Hu (2005), but they use unstable variant of the strain gradient theory with positive sign of the high order term in governing equations (see discussion in Askes and Aifantis, 2009, 2011).

In Papargyri-Beskou et al. (2003) it was analytically shown that non-classical stiffening size effect in a cantilever beam bending problem strongly depends on the length of the gradient beam but not on its thickness. Thus, the behavior of gradient beams models contradicts with widely known models that includes a couple stress effects (see Altenbach & Eremeyev, 2009; Lam et al., 2003; Park & Gao, 2006), where the stiffening size effect is strongly affected by the beam thickness. This result was validated later by the numerical FE simulations in the frame of two-dimensional plane strain statement for a beam three-point bending test in Giannakopoulos, Amanatidou, & Aravas (2006) and for a cantilever beam test in Giannakopoulos and Stamoulis (2007). It was proved the increase of the gradient beam apparent stiffness with the increase of the l/L ratio, where L is a beam length.

Later the similar to the Papargyri-Beskou et al. (2003) approach was used and extended in many papers. Numerical one-dimensional analysis for the gradient beams bending and buckling problems was provided by Tsamasphyros, Markolefas, and Tsouvalas, (2007); Pegios, Papargyri-Beskou, and Beskos (2015) and Challamel et al., (2015). Analytical solutions for several stability problems were given in Lazopoulos and Lazopoulos (2010) and in Artan and Toksöz (2013). Toughness of the notched gradient beams subjected to bending was studied in Stamoulis and Giannakopoulos (2008, 2012). Application of Laplace transform in the gradient beams bending problems was realized in Yayli (2013). Free vibration problems were considered in Yayli (2014). In Polizzotto (2016) it was shown that Euler–Bernoulli model statement is similar for all types of symmetrized Mindlin Form II strain gradient theories. Timoshenko gradient beam model was performed in Polizzotto (2012), and in Triantafyllou and Giannakopoulos (2013). Strain/inertia gradients beams models were given in Askes and Aifantis (2009, 2011); Yaghoubi, Mousavi, and Paavola (2015). It was sown an excellent fit of the molecular-dynamics results for wave dispersion in carbon nanotubes (see Askes & Aifantis, 2009). Identification of material scale parameter of gradient beams models based on simple (structural) molecular dynamics simulations of carbon nanotubes was also done recently in Barretta, Brčić, Čanađija, Luciano, and Marotti de Sciarra (2017). Thermoelastic gradient beam model was presented in Canadija, Barretta and Marotti De Sciarra (2016). Stability of gradient beams on Winkler elastic foundation was studied in Yayli (2017). Comparison between gradient and non-local beam models was presented in Challamel and Wang (2008) and in Challamel (2013). Generalized beam model that includes Eringen's non-local and gradient elasticity models was studied in De Sciarra and Barretta (2014); Li and Hu (2015); Li and Hu (2017); Rajasekaran and Khaniki (2017); Şimşek (2016); Xu et al. (2017). Gradient Kirchhoff plate theory was presented in Papargyri-Beskou & Beskos (2008) and in Papargyri-Beskou, Giannakopoulos, and Beskos (2010). Von Karman's gradient plate model was elaborated in Lazopoulos (2004). Governing equation of this model is equal to (1) in assumption of small deflections and cylindrical bending of the plate. Gradient theory of cylindrical shells was performed in Papargyri-Beskou & Beskos, (2009, 2010) and in Papargyri-Beskou, Tsinopoulos, and Beskos (2012). Strain/inertia gradients model for cylindrical shells was given in Xu and Deng (2015).

Different to the Papargyri-Beskou et al. (2003) approach for the gradient Bernoulli–Euler beam model derivation was proposed in Lazopoulos (2003) and later more systematically in Lazopoulos and Lazopoulos, (2010b). Here the general form of the strain energy density (Mindlin, 1964; Mindlin & Eshel, 1968) and all components of strain, strain-gradient, stress and double stress tensors were analyzed. Classical Bernoulli–Euler hypothesis was used to derive the statement of the beam model. This leads to the high order governing equilibrium equation of the following form:

$$D\, w^{IV} - l^2 D_0 w^{VI} = q, \tag{2}$$