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Elastic fields caused by eigenstrains in two joined half-spaces with an interface of coupled imperfections: Dislocation-like and force-like conditions

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ABSTRACT

This paper presents the derivation of a set of the novel closed-form solutions for the eigenstrain-induced elastic fields in two joined half-space solids with an interface of coupled dislocation-like and force-like imperfects. A model is proposed to characterize the interfacial discontinuity properties concerning displacements and stresses, which permits quantitative jumps of either the displacements or the stresses from one medium to the other. Basic Galerkin vectors in these two imperfectly joined half-spaces are derived based on the boundary conditions, and thus the elastic responses are obtained in explicit expressions. Taking advantage of the three-dimensional fast Fourier Transform algorithms for the convolution and correlation, the elastic fields subjected to one arbitrarily shaped inclusion or more can be efficiently and accurately evaluated. The influences of several types of interfaces, such as the perfectly bonded interface, the dislocation-like interface, and the force-like interface, on the stress and displacement transmissions are discussed. Cases for the elastic fields due to a cuboidal, a spherical, and an array of multiple cuboidal inclusions, are investigated.

1. Introduction

Plastic strain, thermal expansion, phase transformation, or misfits are inevitably encountered in engineering materials. Based on Mura (1982), these strains are all included in the concept of eigenstrains, which is also referred to as the inclusion problems (Eshelby, 1957). Chiu (1977) solved the problem of a cuboidal inclusion in an infinite space, and then utilized the method of mirror image to obtain the stress fields due to a cuboidal inclusion in a half-space with a free surface (Chiu, 1978). Yu and Sanday (1990) solved the stress fields in a half-space with an axisymmetric ellipsoidal inclusion. Jacq, Nelias, Lormand, and Girodin (2002) employed the mirror-image method (Chiu, 1978) to solve a three-dimensional elastic-plastic rolling contact problem, where plasticity was treated as a homogeneous inclusion, or eigenstrain problem. Through the method of Galerkin vectors (Yu & Sanday, 1991a, 1991b) and influence coefficients (Liu & Wang, 2002), Liu and Wang (2005) offered the direct solution to the elastic field caused by an eigenstrian in a half-space material, and then Liu, Jin, Wang, Keer, and Wang (2012) derived a complete set of the closed-form integral kernels for the elastic fields and

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solved the elastic-plastic stress fields in a half-space efficiently and accurately. Combining Eshelby's equivalent inclusion method (EIM) (Eshelby, 1957) and the elastic-plastic contact solutions by Jacq et al. (2002), Koumi, Zhao, Leroux, Chaise, and Nelias (2014), Xie et al. (2014), and Zhou et al. (2014) converted the inhomogeneity problems into inclusion problems, and presented numerical solutions to the stress fields. One may refer to Mura (1982), Mura, Shodja, and Hirose (1996), and Zhou et al. (2013) for detailed reviews of inclusion problems and solutions.

Most of the studies mentioned above were limited to an infinite space or a half-space, whereas in many other cases, inclusions may exist in joined materials, for example, bimaterials, layered materials, materials with coatings, or materials with reinforcements. By using Papkovich–Neuber potentials, Rongved (1955) and Dundurs and Hetényi (1965) obtained Green's functions of the elastic fields, caused by a point force, in perfectly joined or frictionless contacting half-spaces. Considering the complexity of bimaterial Green's functions, Yu and Sanday (1991a, 1991b) employed the Galerkin vectors along with Mindlin's superposition method (Mindlin, 1936), in terms of the Newtonian and the biharmonic potentials, to generalize analytical solutions for the elastic fields in joined half spaces, and then Yu, Sanday, and Rath (1992) solved the elastic fields due to a pure dilatational eigenstrain (thermal inclusion) within a spherical inclusion, in two joined half-spaces either perfectly bonded or in a frictionless contact. Recently, Wang, Yu, and Wang (2016) derived a set of core analytical solutions and proposed an efficient numerical method, by employing the three-dimensional fast Fourier transform algorithm (3D-FFT), to evaluate the elastic field, in two perfectly bonded half-spaces, caused by distributed eigenstrains within inclusions of any shapes. Subsequently, Yu, Wang, and Wang (2016) solved the elastic fields subjected to arbitrary eigenstrains and frictionless contact interface.

However, interfacial defects, such as debonding, microcracks, dislocations, and voids, may appear in the material or component manufacturing processes, or during its life of service, and the mathematic descriptions of these interfaces should be between perfectly bonded and free touching conditions (Sudak & Wang, 2006; Wu, Lv, & Zhang, 2016; Yu, Wei, & Chiang, 2002). Many researchers have explored the influences of interface imperfections on elastic fields, and linear spring-like, dislocation-like, force-like interface models and general interface model have been proposed to characterize these imperfections. In the linear spring-like interface model, it has a thin layer of interphase material and permits continuous tractions across the interface while allowing discontinuous displacements. In addition, the displacements and tractions are coupled, and the jumps in the displacements are linearly proportional to interfacial tractions (Achenbach & Zhu, 1989; Benveniste & Aboudi, 1984; Cheng, Jemah, & Williams, 1996; Hashin, 1991a, 1991b; Jasiuk, Chen, & Thorpe, 1992; Lene & Leguillon, 1982; Levy, 1991; Qu, 1993; Shilkrot & Srolovitz, 1998; Zhong & Meguid, 1996). However, the limit of vanishing layer-thickness in the linear spring-like interface model may lead to physically impossible interpenetration. Yu (1998) developed the dislocation-like interface model without using the layer, in which the displacement and traction are uncoupled. The dislocation-like interface model also permits continuous transmission of interfacial tractions but permits displacements to jump in a linear proportion of a constant at the interface, which was demonstrated through experiments where two photoelastic epoxy resin plates were not perfectly bonded together (Yu et al., 2002). Contrary to the dislocation-like interface model, the force-like interface model allows a jump in the interfacial tractions, instead of displacements (Benveniste & Chen, 2001; Pan, 2003). Wu et al. (2016) utilized the two-dimensional fast Fourier Transform (2D-FFT) algorithms to calculate the elastic field in anisotropic bimaterials at the interface in context of the force-like, dislocation-like and spring-like imperfections. Another interface model, as a member of imperfect interfaces, is the general interface model, where there are jumps in both displacements and tractions at the interface. Javili, Steinmann, and Mosler (2016) employed the computational homogenization, in accordance with the Hill–Mandel condition and the averaging theorem, and investigated the size dependent behavior of materials by consideration of interfaces concerning the discontinuity in displacements and/or tractions (Benveniste, 2013; Hashin, 2002). It should be mentioned that these researches mainly involved thermal inclusions, planar or regular inclusions, such as spherical, cylindrical, or ellipsoidal inclusions. A general closed-form solution for arbitrarily shaped inclusions subjected to an imperfect interface is necessary.

It is possible to develop a theoretical solution set, employing Yu's method (Yu, 1998; Yu & Sanday, 1991a, 1991b), as the analytical core to the solutions of the three-dimensional (3D) elastic fields in two imperfectly joined half spaces, one of which contains an arbitrary inclusion, and the interface imperfection couples the dislocation-like and force-like feature where the jumps in terms of displacements and/or tractions are linearly proportional to jumping coefficients at the interface, one subset of the general interface model (Javili et al., 2016; Wang, Yu, & Wang, 2017). This is one of the goals of the work reported in this paper. On basis of the analytical results of the coupled imperfect-interface models, an efficient semi-analytical solution approach is proposed to solve the elastic fields numerically by implementing the three-dimensional fast Fourier Transform (3D-FFT) algorithms (Liu & Wang, 2002; Liu et al., 2012; Liu, Wang, & Liu, 2000); and this is the other target of the reported research.

2. Theory

Fig. 1 illustrates the proposed model for two imperfectly joined isotropic elastic half-spaces in the Cartesian coordinate system, $Ox_1x_2x_3$, which consists of half-space I ($x_3 \geq 0$) with shear modulus and Poisson's ratio ν, and half-space II ($x_3 \leq 0$) with shear modulus μ' and Poisson's ratio ν'. A coupled interface model permitting jumps in either the displacements or stresses across the interface ($x_3 = 0$) is needed to evaluate elastic μ fields induced by inclusions arbitrarily distributed in one of them, for example, half-space I. This interface model should allow the perfectly bonded interface (Wang et al., 2016; Yu & Sanday, 1991a), the force-like and/or the dislocation-like imperfection (Yu, 1998; Yu et al., 2002). In the two dissimilar