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Nonlocal nonlinear analysis of functionally graded plates using third-order shear deformation theory

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ABSTRACT

In this work, nonlocal nonlinear analysis of functionally graded plates subjected to static loads is studied. The nonlocal nonlinear formulation is developed based on the third-order shear deformation theory (TSDT) of Reddy (1984, 2004). The von Kármán nonlinear strains are used and the governing equations of the TSDT are derived accounting for Eringen's nonlocal stress-gradient model (Eringen, 1998). The nonlinear displacement finite element model of the resulting governing equations is developed, and Newton's iterative procedure is used for the solution of nonlinear algebraic equations. The mechanical properties of functionally graded plate are assumed to vary continuously through the thickness and obey a power-law distribution of the volume fraction of the constituents. The variation of the volume fractions through the thickness have been computed using two different homogenization techniques, namely, the rule of mixtures and the Mori–Tanaka scheme. A detailed parametric study to show the effect of side-to-thickness ratio, power-law index, and nonlocal parameter on the load-deflection characteristics of plates have been presented. The stress results are compared with the first-order shear deformation theory (FSDT) to show the accuracy of nonlocal nonlinear TSDT formulation.

1. Introduction

Functionally graded materials (FGMs) are the special class of composites, in which the volume fraction of two or more materials are varied continuously as a function of position along certain dimension(s) of the structure to achieve a required functionality. The concept of FGMs was proposed by materials scientists, based on its use in applications such as thermal barrier materials (Koizumi, 1993; 1997). FGMs have many other applications such as Bio materials (Pompe et al., 2003), Dental implants (Watari, Yokoyama, Saso, & Kawasaki, 1997), Sensors, Thermo-generators (Aller, Drar, Schilz, & Kaysser, 2003) and wear resistant coatings (Schulz, Peters, Bach, & Tegeder, 2003). Due to smooth and continuous variation of material properties from one surface to the other, FGMs are usually superior to the conventional composite materials. FGMs possess a number of advantages, including a reduction of in-plane and transverse through-the-thickness stresses, an improved residual stress distribution, enhanced thermal properties, higher fracture toughness, and reduced stress intensity factors along with high wear resistance. Therefore, accurate determination of the deformation and stress variation in such structures is important. FGMs are mostly used in nano/micro components, in which it is necessary to account for small scale

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effects, higher-order kinematics, and geometric nonlinearity. In view of this, the present work accounts for non locality, nonlinearity and higher-order kinematics through the TSDT (Reddy, 1984; 2004).

Micro structural and size effects play a dominant role when the structures such as nano beams made up of FGMs are used for structural purposes (Rahmani & Pedram, 2014; Simsek & Yurtcu, 2013). It has been observed that, the boundary value problems solved using a classical continuum mechanics approach cease to give solutions that are comparable to experimental solutions in case of micro/nano structures. The main reason for this discrepancy is that the classical continuum approach lacks an internal length scale that takes care of material micro structure. Many researchers have discussed the importance of size effects in nano/micro structures (John, George, & Richard, 2003; Kröner, 1967; Krumhansl, 1968; Kunin, 1984). Nonlocal continuum models are found to be superior to their local counterparts for the analysis of nano structures (Arash & Wang, 2012; Edelen & Laws, 1971). Many works discussed the importance of nonlocal theories in the analysis of beams and plates (Reddy, 2007a; 2010; Roque, Ferreira, & Reddy, 2011; Thai, 2012). The effect of non locality appears in the constitutive relations via a length scale parameter. These constitutive relations are also proven to avoid the singularity at the crack tip in Fracture mechanics (Zhou, Han, & Du, 1999). These models are also reported to achieve properly convergent solutions for localized damage analysis (Bažant & Milan, 2002). In the last two decades, Eringen's nonlocal theory (Eringen, 1972; 1983; Eringen & Edelen, 1972) has received significant attention. Eringen's stress gradient model is based on the assumption that the stress at a point is a function of the strain field at all neighboring points on the continuum. The interatomic forces and atomic length scales directly come in the constitutive equations as material parameters (Eringen, 2002). Recently, Eringen's model has been modified using the gradient elastic model as well as an integral non-local elastic model that is based on combining the local and the non-local curvatures in the constitutive elastic relations (Challamel & Wang, 2008; Fernández-Sáez, Zaeraa, Loyaa, & Reddy, 2016; Romano, Barretta, Diaco, & de Sciarra, 2017).

Modeling of material through the thickness for FGMs has been important for accurate analysis of FGMs. Some works on the variation of material properties through the thickness according to a power-law distribution and the locally effective material properties in terms of the volume fractions of the constituents through the Mori–Tanaka scheme includes Reddy and Cheng (2001) and Vel and Batra (2002). Kashtalyan (2004) derived the three-dimensional elasticity solution for a functionally graded simply supported plate subjected to transverse loading. A three-dimensional solution for the problem of clamped rectangular plates of arbitrary thickness is presented by Elishakoff and Gentilini (2005). Aghababaei and Reddy (2009) provided analytical solutions of bending and free vibration of plates using the nonlocal TSDT. Jandaghian and Rahmani (2016) provided analytical solutions of vibration analysis of functionally graded piezoelectric nano plates based on Eringen's non local theory and Kirchhoff plate theory. Analytical and finite element models of functionally graded plates using first order theory and third-order theory was presented by Reddy (2000). Many researchers have recently attempted to study the behavior of FGM plates using higher order theories (Aliaga & Reddy, 2004; Golmakani & Kadkhodayan, 2011; Reddy & Kim, 2012; Talha & Singh, 2010). Reddy (2010) presented the formulation for FGM plates considering a general third-order theory and the von Kármán nonlinear strains. Kim and Reddy (2015) presented the theory and finite element analysis of functionally graded plates with modified couple stress effect and the von Kármán nonlinearity. Mousavi, Paavola, and Reddy (2015) presented a variational approach based on Hamilton principle and developed the governing equations for the dynamic analysis of plates using the Reddy third-order shear deformable plate theory accounting for the strain gradient and velocity gradient effects. Ferreira, Batra, Rouque, Qian, and Martins (2005) and Qian, Batra, and Chen (2004) carried out analysis of FGM plates using higher order theories and meshless methods. Transient, thermo-elastic, bending and vibration analysis of the functionally graded plates using FSDT were presented by many researchers such as Praveen and Reddy (1998), Singha, Prakash, and Ganapathi (2011), Hosseini-Hashemi, Taher, Akhavan, and Omidi (2010), Reddy and Chin (1998). Reddy, El-Borgi, and Romanoff (2014) performed the nonlinear analysis of functionally graded micro beams using Eringen's nonlocal differential model. Rahaeifard, Kahrobaiyan, Ahmadian, and Firoozbakhsh (2013) applied strain gradient formulation to investigate the effect of length scale on the static and dynamic behavior of Euler–Bernoulli beams made of functionally graded materials. Salehipour, Shahidi, and Nahvi (2015) formulated a novel nonlocal theory that assumes the nonlocal strain at a point as a function of local strain at all neighboring points. This novel theory was applied to study the functionally graded plates using the first order plate theory and the results are validated with Eringen's three dimensional nonlocal models. Nejad and Hadi (2016) investigated bi-directional functionally graded Euler–Bernoulli nano beams subjected bending using Eringen's non-local elasticity theory. Recently Ghayesh, Farokhi, Gholipour, and Tavallaeinejad (2017) investigated the effect of the small scale parameter, the gradient index on the nonlinear behavior of functionally graded tapered beams subjected to bending and forced vibration. Here they used a modified couple stress theory to account for the size dependent behavior of the functionally graded material. Raghu, Rajagopal, and Reddy (2018) developed nonlinear finite element model using Eringen's nonlocal model and von Kármán nonlinear strains to analyze the laminated composite plates using Reddy's third-order shear deformation theory (Reddy, 1984).

In this paper, the formulation for nonlocal nonlinear analysis of FGM plates is presented. The behavior of FGM plates subjected to static loads is studied. The Reddy TSDT with the von Kármán nonlinear strains is used for deriving the governing equations that accounts for Eringen's nonlocal stress-gradient model. The nonlinear displacement finite element model is developed from the resulting nonlocal nonlinear equations. Two homogenization techniques, namely rule of mixtures and Mori–Tanaka scheme is used to find the mechanical properties of the FGM plate using power-law distribution of the volume fraction of the constituents. The presented results are compared with the literature and the percentage difference between the deflections obtained using two techniques are tabulated in the results. The effect of side-to-thickness ratio, power-law