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Rotational waves in layered solids with many sliding layers

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ABSTRACT

Wave propagation in layered materials with free sliding layers is analysed in 2D using anisotropic Cosserat continuum formulation, which accounts for layer bending. It is found that the 2D Cosserat continuum supports propagation of waves of three types: usual longitudinal and transverse (shear) waves and a new one – the rotational wave. The velocities of the longitudinal and transverse waves strongly depend upon the direction of their propagation with respect to the layering, but exhibit only marginal dispersion. Opposite to this the rotational wave is strongly dispersive: its velocity increases proportionally to the wavelength, but marginally depends upon the propagation direction. A prominent feature of the rotational wave is its extremely high velocity that can be considerably greater than the velocity of longitudinal wave propagating in the material of the layers. This high velocity can be used to distinguish the rotational waves. Furthermore, this property allows detecting areas of sliding, which is important in both materials engineering for non-destructive control and in geoscience for monitoring the state of the Earth's crust.

1. Introduction

Layered solids – micro-structured materials of a special kind with inherent anisotropy – are often encountered in both engineering (e.g., various layered composite materials) and nature (e.g., layered and stratified rocks and rock masses). Thus efficient modelling of such materials is of great importance for many areas of Engineering and Earth sciences. The modelling also presents considerable challenge. The first, albeit minor complication in the modelling of layered materials comes from the fact that even if the layers are isotropic and the properties change in one direction only, the layered material possesses anisotropy (transverse isotropy). Further complication occurs when the behaviour of interfaces between the layers needs to be taken into account. Especially interesting is the case when the layers can freely slide; this is the case considered in this paper.

While explicit modelling of layered materials (that is writing down the field equations for every layer, then satisfying the transition conditions on their interfaces and free surfaces, e.g., Boudouti El, Djafari-Rouhani, Akjouj, & Dobrzynski, 2009; Brechovskich, 1980; Lewick, Burridge, & de Hoop, 1996; Salamon, 1991) may look as an ultimate tool, the efficiency is quickly lost when the layers are very thin compared to the size of the area to be modelled. Then explicit numerical modelling (e.g., by the finite element method) calls for very fine discretisation leading to large numbers of elements and requiring significant computational resources, especially when dynamic effects and wave propagation are of interest. That is why in the case of many thin layers the use of continuum methods based on a proper homogenisation is attractive.

Consider a layered material with layers thickness (in general an average or characteristic layer thickness) b, Fig. 1. A rational approach to homogenisation (see also Pasternak & Mühlhaus, 2005a for review of homogenisation methods) is based

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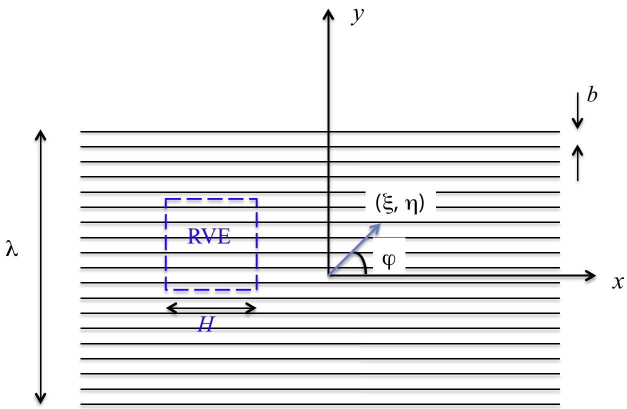

Fig. 1. Layered material with layer thickness b, external size or wavelength λ and the representative volume element RVE. Vector (ξ, η) in co-ordinate frame (x, y) is a 2D wave vector.

on the so-called hypothesis of separation of scales. The approach presumes that a representative volume element (RVE) of size, H, can be introduced satisfying the following double inequality (e.g., Batchelor, 1974; Hunter, 1976; Krajcinovic, 1996; Scipio, 1967):

$$b << H << l \tag{1}$$

where λ is a characteristic length of the variations of the external fields, which is either an external characteristic length (as shown in Fig. 1) or a wavelength. The later will be used here as the external characteristic length for the investigation of wave propagation in layered materials.

The RVE plays a central role in the homogenisation procedure as the equivalent continuum is constructed based on the stress and strain fields averaged over the RVE (in the sense of moving average), leading to what is often referred to as the macroscopic fields. The macroscopic fields are generally smoother than the original (microscopic) fields. In the process of averaging the equations of equilibrium (or motion), compatibility and the constitutive equations are obtained. These describe a continuum that models macroscopic (with respect to the layer thickness) behaviour of the layered material. One needs to emphasise that this equivalent continuum is only a model of the layered material.

If the obtained equations represent a classical continuum (elastic or inelastic) then they do not explicitly contain layer thickness (as the constitutive and equilibrium equations do not contain any characteristic lengths). Furthermore, these equations are point-wise and hence do not contain characteristic size λ either. Therefore the hypothesis of separation of scales, that is double inequality (1) translates into the following double asymptotics

$$\frac{b}{H} \to 0, \ \frac{H}{\lambda} \to 0 \tag{2}$$

of which the classical continuum retains only the main terms, that is terms that do not possess characteristic lengths b and λ. Retaining higher terms in the first asymptotics (2) will lead to generalised continua such as the Cosserat continuum, gradient continua, etc. (see e.g., Dyskin & Pasternak, 2015; Pasternak, 2002; Pasternak & Mühlhaus, 2005a for details).

The most challenging part in the homogenisation of materials with microstructure is formulating the constitutive equations, which involves the determination of the effective characteristics (e.g., effective moduli or compliances) relating the macroscopic (averaged over RVE) stress and strain. In this respect the layered materials present a simple enough case when the layer properties change in only one direction, such that the homogenisation procedure is one-dimensional. Especially simple is the case when the layers are connected in elastic manner without any possibility of sliding. The homogenisation then produces close form solutions even for arbitrary anisotropy of the layers (e.g., Gerrard, 1982; Lifshitz & Rosenzweig, 1946, 1951; Salamon, 1968). Layering of course changes anisotropy, for instance if the layers are isotropic the equivalent elastic continuum becomes transversely isotropic.

When sliding between the layers is allowed the situation becomes complex, since sliding makes the effective continuum non-elastic; if friction is present and the friction coefficient is constant the equivalent continuum becomes elastoplastic. Furthermore, non-elasticity can lead to localisation of plastic zones leading to emerging new characteristic lengths, which would complicate homogenisation. Nevertheless there exists a simple case – the case of absence of friction such that sliding always occurs. This special case presents a good model that would allow a simplified analysis of wave propagation and could provide insight into the behaviour of more realistic materials with friction.

Hereafter, for the sake of simplicity, we consider a material made of equal isotropic layers of thickness b, Fig. 1, capable of free sliding. It will also be assumed that the layered material is under compression in the direction normal to layers such that the layers are kept together. Furthermore, it will be assumed that no delamination is possible such that layers are always in contact. The simplest model of such a material is an anisotropic (transversely isotropic in this case) elastic