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On the mechanics of Kirchhoff and Mindlin plates incorporating surface energy

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ABSTRACT

In this paper, size-dependent Kirchhoff and Mindlin plate models are developed to investigate the coupling effects of nonlocal stress, strain gradient and surface energy on the dynamic response of nanoplate. The nonlocal stress and strain gradient effects are captured by nonlocal strain gradient theory, while the surface energy effects are incorporated by surface elasticity theory. The governing equations of motion and related boundary conditions are derived from Hamilton's principle. Analytical solutions for the natural frequencies of simply supported nanoplate are obtained through the Navier approach. A good agreement between the results of the present models and those available in literatures are observed. Selected examples are presented to show the influences of nonlocal parameter, material length scale parameter, length-to-thickness ratio, aspect ratio, surface energy and shear deformation on the vibration behavior of nanoplate. It is found that for nanoplate with lower length-to-thickness ratio, the nonlocal stress, strain gradient and surface energy have remarkable influences on the vibration characteristic simultaneously. While surface effects play a dominant role in the vibration behavior for nanoplate with higher length-to-thickness ratio.

1. Introduction

Nanostructures are the key elements of nano-electromechanical systems (NEMS) (Craighead, 2000; Ekinci & Roukes, 2005) due to their extraordinary mechanical, electrical and thermal properties. In recent years, they have been widely used to design and manufacture the small-scaled devices, such as nano-sensors (Robinson, Perkins, Snow, Wei, & Sheehan, 2008), nano-resonators (Chen et al., 2009), nano-actuators (Park, An, Suk, & Ruoff, 2010). However, significant size effects have been experimentally observed in such applications. It is known that classical continuum mechanics cannot capture the size effects due to lack of material length scale parameters. To understand the size-dependent mechanical characteristics of nanostructures, many non-classical continuum theories have been proposed, such as nonlocal elasticity theory (Erigen, 1972, 1983), strain gradient theory (Mindlin, 1964, 1965), surface elasticity theory (Gurtin & Murdoch, 1975, 1978), and nonlocal strain gradient theory (Lim, Zhang, & Reddy, 2015). Among these, the nonlocal strain gradient theory have gained much attention by the researchers recently.

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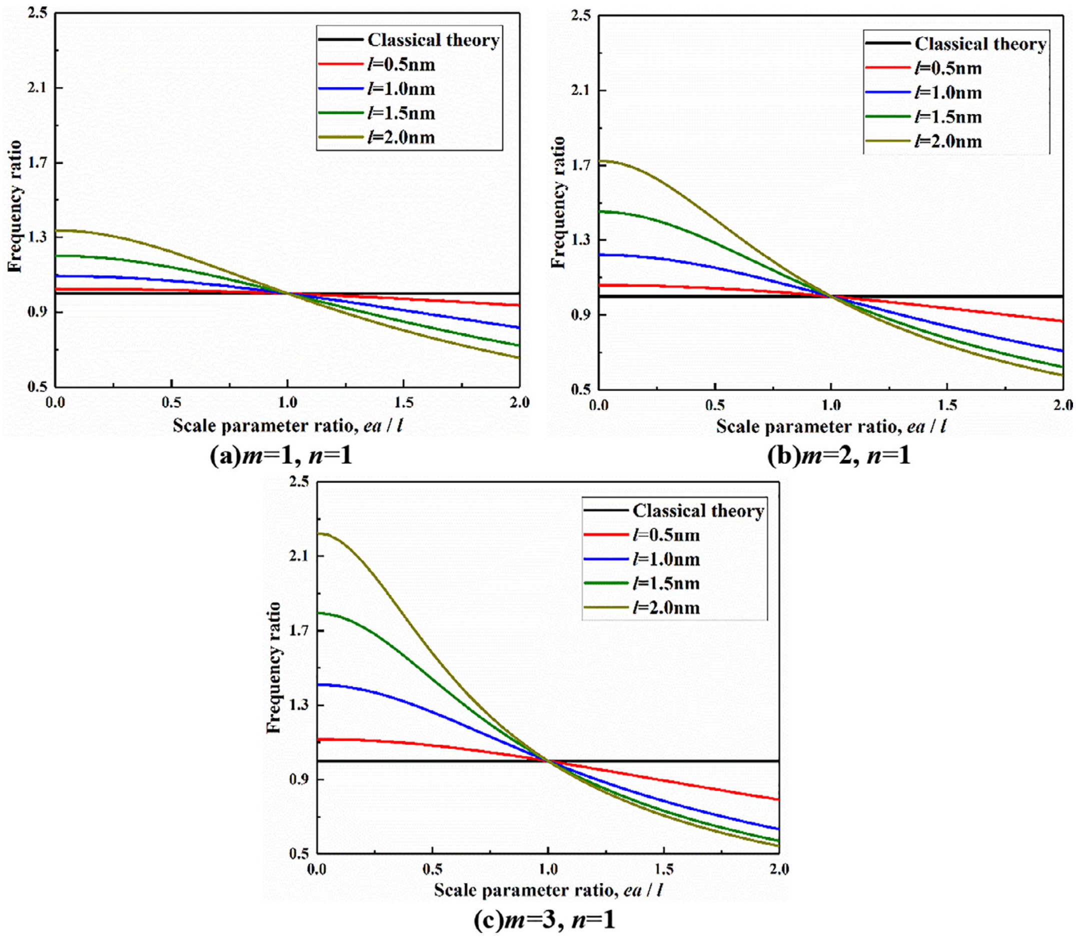

Fig. 1. Variation of frequency ratio with respect to scale parameter ratio corresponding to different material length scale parameters ($l_a/h = 10$, $l_a/l_b = 1$).

Nonlocal strain gradient theory (Lim et al., 2015) is a newly developed non-classical continuum theory, which can be considered as a combination of nonlocal elasticity theory and strain gradient theory. In the nonlocal strain gradient theory, a nonlocal parameter is introduced to evaluate the importance of nonlocal effect and a material length scale parameter is involved to access the remarkable strain gradient effect. Unlike the nonlocal elasticity theory or strain gradient theory can only capture the stiffness-softening or stiffness-hardening effect, the nonlocal strain gradient theory achieves the possibility of capturing both the stiffness-softening and stiffness-hardening effects by adjusting the two scale parameters. Recently, on the basis of nonlocal strain gradient theory, numerous works have been performed to study the size-dependent mechanical behaviors of nanostructures, such as nanobeams (Ebrahimi & Barati, 2017a,b; Li & Hu, 2015, 2016, 2017; Li et al., 2017; Li, Li, & Hu, 2016; Lu, Guo, & Zhao, 2017a; Rajasekaran & Khaniki, 2017; Sahmani & Aghdam, 2017a; Simsek, 2016; Xu, Wang, Zheng, & Ma, 2017; Yang, Yang, & Wang, 2016), nanorods (Li, Hu, & Li, 2016; Xu, Zheng, & Wang, 2017; Zhu & Li, 2017a), nanoshells (Mahinzare, Mohammadi, Ghadiri, & Rajabpour, 2017; Sahmani & Aghdam, 2017b,c; Zeighampour, Beni, & Karimipour, 2017), and nanoplates (Barati & Shahverdi, 2017a; Ebrahimi & Barati, 2017c; Ebrahimi & Dabbagh, 2017; Ebrahimi, Barati, & Dabbagh, 2016; Farajpour, Yazdi, Rastgoo, & Mohammadi, 2016). For example, Lu, Guo, and Zhao (2017b) proposed a unified size-dependent higher-order beam model to analyze the bending and buckling behaviors of nanobeams via nonlocal strain gradient theory. Zhu and Li (2017b) developed a size-dependent integral elasticity model for small-scaled rod in tension based on the nonlocal strain gradient theory. In another work, Shahverdi and Barati (2017) investigated the vibration of a porous functionally graded nanoplate subjected to hygro-thermal loading by using nonlocal strain gradient theory and four-variable plate theory. Sahmani and Aghdam (2018) employed nonlocal strain gradient theory to study the size-dependent nonlinear buckling and postbuckling characteristics of magneto-electro-elastic composite nanoshells. These works have shown that both nonlocal effect and strain gradient effect have a remarkable influence on the mechanical response of nanostructures.

On the other hand, one of the main features of nanostructures is their high surface-to-bulk ratio, which makes the elastic response of their surface layers to be quite different from bulk parts and causes to exhibit different behavior compared to the conventional structures. To demonstrate the significant influence of surface effects on the elastic responses