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The transient response of multiferroic composites

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ABSTRACT

The transient response of multiferroic magnetoelectric composite materials which consist of electro-magneto-thermo-elastic phases is determined. This response is governed by the fully coupled constitutive relations, according to which the electro-magneto-elastic field affects the temperature and vice versa, as well as the dynamic equations of motion, Maxwell and energy. The method of solution is based on the discretization of the composite domain into subcells in everyone of which the governing equations are imposed in the average (integral) sense. The interfacial and boundary conditions are also imposed at the average sense. The validity of the offered method of solution is examined in four special cases where analytical solutions can be established. The proposed approach is implemented on a three ceramic layered composite strip whose transient response is determined under various types of time-dependent loading. Comparisons between the responses based on the transient and quasi-static solutions are shown.

1. Introduction

Multiferroic composites exhibit magneto-electro-thermo-elastic behavior caused by its piezoelectric and piezomagnetic phases. As a result the application of electric field on the composite, the piezoelectric phase deforms and the resulting mechanical strain induces magnetic effects in the piezomagnetic constituent. Similarly, the application of a magnetic field causes the deformation of the piezomagnetic phase which induces electric polarization in the piezoelectric constituent. Due to the mechanical deformation which exist in both its piezoelectric and piezomagnetic phases, multiferroic composites can be utilized as sensors, actuators and transducers. The remarkable effects caused by electric, magnetic, thermal and mechanical field coupling in multiferroic composites are not present of course in composites that consist of a single piezoelectric or piezomagnetic phase. For reviews of multiferroic composites including their structure, properties and applications, see Nan, Bichurin, Dong, Viehland, and Srinivasan (2008), Ma, Hu, Li, and Nan (2011), Ortega, Kumar, Scott, and Katiyar (2015) and Palneedi, Annapureddy, Priya, and Ryu (2016), for example. In the third and fourth articles lists of some piezoelectric, magnetic and other types of materials which can be used as constituents in multiferroic composites are given. Multiferroic composites can be designed in the form of particulate, fibrous and layered composites.

There are numerous articles devoted for the prediction of the effective properties of linear electro-magneto-thermo-elastic composites by employing various types of micromechanical analyses, see Carman, Cheung, and Wang (1995), Li and Dunn (1998), Li (2000), T-L and Huang (2000), Huang, Liu, and W-L (2000), Aboudi (2001), Lee, Boyd, and Lagoudas (2005), Tang and Yu (2009), Bravo-Castillero et al. (2009), Challagulla and Georgiades (2011) and Kim (2011), for example. These micromechanical analyses predict, in particular, the magneto-electric coupling coefficients which are of course absent in the material characterization of the monolithic constituent. Thus, Huang et al. (2000) established the optimum fiber volume

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ratios at which the magneto-elastic coupling coefficients reach their maximum values. A multiscale formulation for the analysis of the one-way coupled heat conduction with electromechanical nonlinear deformation in heterogeneous bodies has been presented by Muliana and Lin (2011). A micromechanical analysis of multiferroic composites which are composed of magnetostrictive and piezoelectric constituents have been presented by Jin and Aboudi (2015).

Dynamic effects in the form of harmonic wave propagation in linear electro-magneto-elastic materials have been considered by several investigators, see for example Melkumyan (2007), Wu, Shen, and Sun (2007), Pang, Liu, Wang, and Zhao (2008) and Zhang, Pang, and Feng (2014) and references cited there.

The analysis of transient effects in linear electro-magneto-thermo-elastic materials is far more difficult. Such effects may occur when an electro-magneto-thermo-elastic devise is subjected to a sudden application of mechanical loading, or the abrupt application or cutting off an applied electric field. In Ootao and Tanigawa (2005), the authors investigated the transient response of a multilayered magneto-electro-thermo-elastic strip due to applied temperature at the surfaces. In this article however the mechanical deformation is governed by the equilibrium rather than the dynamic equations of motion and only one-way thermal effects have been considered i.e., the temperature affects the electro-magneto-elastic fields but not the other way.

In the present investigation a method of solution for establishing the transient response of multiferroic composites is presented. To this end, full thermo-electro-magneto-elastic coupling is considered in which the thermal, electric, magnetic and elastic field interact and affect each other. The mechanical deformation and temperature are governed by the equations of motion and coupled energy, whereas the electric and magnetic fields are governed by the steady state Maxwell equations, in conjunction with the quasi-electrostatic and quasi-magnetostatic approximations. The fully coupled constitutive equations are derived by employing the energy balance, the Clausius–Duhem inequality and a first order expansion of the Gibbs potential, from which the expressions for the stresses, electric displacements, magnetic inductions and entropy are established.

The present method of solution forms a generalization of the analysis of functionally graded materials that has been discussed in Chapter 11 of Aboudi, Arnold, and Bednarcyk (2013), and whose accurate and reliable prediction are discussed. It is based on discretizing the composite into a number of regions (subcells), every one of which contains a distinct homogeneous material. The volume-integrals of the governing equations of motion, Maxwell and energy, are satisfied. As to the various interfacial conditions between the internal subcells and boundary conditions at the external ones, they are imposed in the average (integral) sense.

The present solution approach is verified in the following four special cases where analytical solutions exist.

(1) The transient response to mechanical loading of elastic half-space.
(2) The transient solution of the coupled thermoelastic equations which govern the response of a half-space to thermal loading.
(3) The steady state solution of the coupled elastic and Maxwell equations.
(4) The steady state solution of the coupled elastic, Maxwell and heat equations.

The offered method of solution approach is implemented on a composite which consists of two ceramic barium titanate ($BaTiO_3$) piezoelectric layers between which a ceramic cobalt ferrite ($CoFe_2O_4$) piezomagnetic layer is located. This electro-magneto-thermo-elastic layered composite is first subjected to a mechanical time-dependent loading, and then to a combined mechanical, electric and magnetic loading. In both cases, the time-dependent response of the composite is shown and discuss. Furthermore, comparisons between the responses based on the transient and quasi-static solutions in which the inertia terms in the equations of motion are deleted are shown.

This article is organized as follows. The Section 2, the fully coupled constitutive relations and governing equations are established. The method of solution is discussed in Section 3 where the mechanical, electromagnetic and thermal equations are discussed. Furthermore, the method of solution of the quasi-static equations which cannot be obtained as a special case from the dynamic analysis is also presented in this section. The verifications of the offered approach are presented in Section 4, which is followed by the Application and a Conclusion sections.

2. Constitutive and governing equations

The constitutive equations that govern the behavior of thermo-electro-magneto-elastic material can be established by generalizing the thermo-piezoelectric equations of Patron and Kudryavtesev (1988) and Kuang (2013) by incorporating the magnetic effects. This generalization is briefly summarized in the following.

The internal energy U can be expressed in terms of the strain components ϵ_{ij}, electric displacements D_i, magnetic inductions B_i and entropy density η, and its rate is given as follows

$$\dot{U} = \frac{\partial U}{\partial \epsilon_{ij}}\dot{\epsilon}_{ij} + \frac{\partial U}{\partial D_i}\dot{D}_i + \frac{\partial U}{\partial B_i}\dot{B}_i + \frac{\partial U}{\partial \eta}\dot{\eta} \tag{1}$$

It can be shown that the local energy equation is given by

$$\dot{U} = \sigma_{ij}\dot{\epsilon}_{ij} + E_i\dot{D}_i + H_i\dot{B}_i + W - \frac{\partial q_i}{\partial x_i} \tag{2}$$