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Dynamics of functionally graded viscoelastic microbeams

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A R T I C L E I N F O

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A B S T R A C T

In this paper, a size-dependent continuum-based model is presented for the coupled nonlinear dynamics of extensible functionally graded (FG) microbeams with viscoelastic properties. The length-scale effect is incorporated based on the modified couple stress theory (MCST). Moreover, employing the Kelvin–Voigt viscoelastic model, viscous components are taken into consideration in both stress and deviatoric segments of the symmetric couple stress tensors. The variation of the material properties of the FG viscoelastic microbeam along the thickness is approximated with the use of the Mori–Tanaka homogenisation method. Both the transverse and longitudinal motion as well as inertial terms are included in the size-dependent continuum model and numerical calculations. The elastic potential energy, kinetic energy and the viscos work are obtained with the consideration of size effects. Using von Karman's strain-displacement relations together with Hamilton's principle, the coupled differential equations of motion are derived. Then, Galerkin's approach and a continuation technique are used in order to obtain the fundamental frequency and dynamic response of the FG viscoelastic microbeam. The effects of parameters such as the gradient index, excitation frequency, the amplitude of the harmonic load and viscoelastic parameters on the nonlinear frequency- and force-responses of the FG viscoelastic microbeams are investigated in details.

1. Introduction

Functionally graded (FG) materials (Ghayesh, Farokhi, Gholipour, & Tavallaeinejad, 2018) have attracted an extensive attention in many engineering fields due to their interesting mechanical properties. The material characteristics of a FG structure vary continuously along one or two specific directions, and this makes them desirable for many applications. A typical FG structure is usually composed of ceramics and metal which can stand high temperatures and mechanical loads, respectively. In the last decade, the development of advanced manufacturing methods has made it possible to produce FGM microstructures such as microbeams and microplates which has the potential to be used in microelectromechanical systems (MEMS) devices (Lü, Lim, & Chen, 2009; Witvrouw & Mehta, 2005).

Experimental observations on micro/nanoscale structures (Fleck, Muller, Ashby, & Hutchinson, 1994; McFarland & Colton, 2005) have shown that these structures exhibit size-dependent mechanical characteristics which cannot be seen through use of the classical elasticity theories. Hence, modified elasticity theories (Dehrouyeh-Semnani, Dehrouyeh, Torabi-Kafshgari, & Nikkhah-Bahrami, 2015; Farokhi & Ghayesh, 2015a,b; Farokhi, Ghayesh, & Amabili, 2013a; Ghayesh & Amabili, 2014; Ghayesh & Farokhi, 2015a; Ghayesh, Farokhi, & Amabili, 2013a; Ghayesh, Farokhi, & Amabili, 2014; Ghayesh, Farokhi, & Hussain, 2016; Gholipour et al., 2015; Li & Pan, 2015; Tang, Ni, Wang, Luo, & Wang, 2014) such as the nonlocal continuum mechanics (Farajpour & Rastgoo, 2017; Farajpour, Rastgoo, & Farajpour, 2017; Farajpour, Shahidi, Mohammadi, & Mahzoon, 2012),

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modified couple stress, nonlocal strain gradient theory and strain gradient theory (Ghayesh, Amabili, & Farokhi, 2013a; Lazopoulos & Lazopoulos, 2010) are widely used in order to analyse the dynamic and static responses of micro/nanoscale structures (Asghari, Kahrobaiyan, & Ahmadian, 2010; Baghani, 2012; Dehrouyeh-Semnani, 2014; Dehrouyeh-Semnani, BehboodiJouybari, & Dehrouyeh, 2016; Farokhi, Ghayesh & Gholipour, 2017; Farokhi, Ghayesh, Gholipour, & Tavallaeinejad, 2017b; Farokhi, Ghayesh, & Amabili, 2013b; Farokhi, Ghayesh, Gholipour, & Hussain, 2017; Farokhi, Ghayesh, Gholipour, & Tavallaeinejad, 2017a; Ghayesh & Farokhi, 2017a; Ghayesh & Farokhi, 2015b; Ghayesh & Farokhi, 2017b; Ghayesh, Farokhi, & Gholipour, 2017a, c; Ghayesh, Amabili, & Farokhi, 2013b; Ghayesh, Farokhi, & Amabili, 2013b; Ghayesh, Farokhi, & Gholipour, 2017b; Hosseini & Bahaadini, 2016; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Karparvarfard, Asghari, & Vatankhah, 2015; Kong, Zhou, Nie, & Wang, 2008; Mojahedi & Rahaeifard, 2016; Taati, 2016). In this work, we will apply the MCST (Farokhi & Ghayesh, 2016; Ghayesh, Farokhi, & Amabili, 2013c), which is capable of predicting the size effects properly, is employed.

In recent years, the linear size-dependent vibrations, bending and buckling of FG microbeams and microplates have been investigated in the literature. Ke, Yang, Kitipornchai, and Bradford (2012) examined the buckling, vibrations, and bending of annular microscale plates made of FG materials using the MCST and the Mindlin plate theory. In their work, the material properties of the FG microplates were varied in the thickness direction. Based on the MCST, Thai, Vo, Nguyen, and Lee (2015) studied the bending, buckling and free dynamics of FG sandwich microbeams. In another interesting work, Lei, He, Zhang, Gan, and Zeng (2013) used the strain gradient elasticity theory to examine the bending and free vibrations of a FG sinusoidal microbeam. Furthermore, a size-dependent continuum model was developed for the bending and dynamic behaviours of microbeams made of FG materials based on the Timoshenko theory (Tajalli et al., 2013). With the aid of the strain gradient theory and Kirchhoffs plate theory, an analytical approach was presented to analyse the buckling behaviour of a multi-microplate structure subjected to biaxial compressive loads and mad of orthotropic materials (Hosseini, Bahreman, & Jamalpoor, 2016). More recently, based on Hamilton's principle, the linear vibrations of beams made of bi-directional FG materials with size effects were examined by Nejad, Amin, and Ali (2017). The size-dependent bending behaviour of FG piezoelectric microplates under a thermo-mechanical load and external electric voltage was studied by Abazid and Sobhy (2017). All these interesting investigations are limited to the linear bending, buckling or vibrations of microscale structures. In a few recent papers (Ghayesh, 2017; Ghayesh & Farokhi, 2017c; Ke, Wang, Yang, & Kitipornchai, 2012), geometric-type nonlinearities were taken into consideration. For example, Reddy, El-Borgi, and Romanoff (2014) investigated the nonlinear response of FG microbeams based on Eringen's model. They used the virtual work principle and the finite element method in order to derive and solve the nonlinear differential equations of FG microbeams with various boundary conditions. Ghayesh, Farokhi, and Gholipour (2017d) investigated the nonlinear forced dynamic response of FG Timoshenko microscale beams in the frame of the MCST.

In the present work, the forced nonlinear dynamics of functionally graded microbeams with viscoelastic properties is studied for the first time. In more details, to authors' best of knowledge, from the open literature, it can be concluded that the coupled longitudinal-transverse nonlinear dynamics of FG viscoelastic microbeams subjected to external force with consideration of size effects has not been investigated yet. This motivates to present a non-classical continuum model for the aforementioned problem in this paper. The MCST and Hamilton's principle are employed to formulate the motion of the microbeam considering the transverse and axial displacements in addition to the inertia. The properties of the FG viscoelastic microbeam are described based on the Mori–Tanaka homogenisation method. The coupled governing equations are discretised with the aid of the Galerkin approach and then a numerical solution is applied employing a continuation method. Finally, the influence of various parameters including viscoelastic linear and nonlinear damping coefficients on the dynamic behaviour is discussed.

2. System modelling and method of solution

Fig. 1 shows an extensible functionally graded viscoelastic microbeam; a distributed harmonic force excites the microbeam in the transverse direction. The length, thickness, width and cross-sectional area are denoted by L, h, b and A, respectively. As shown in the figure, the transverse harmonic load is considered as $F(x)\cos(\omega t)$ where F and ω are respectively the magnitude of the external load and the excitation frequency. Moreover, x and t represent the axial coordinate and time, respectively. The microscale beam is composed of ceramics and metal with material properties which vary in the thickness direction. The lower part of the FG viscoelastic microbeam is mainly made of ceramics, while the upper part chiefly consists of metal. As a homogenisation method, the Mori–Tanaka scheme is used; the effective values of bulk modulus K and shear modulus μ of FGMs can be written as

$$\begin{aligned}\frac{\mu-\mu_m}{\mu_c-\mu_m}&=\frac{\upsilon_c}{1+\upsilon_m(\mu_c-\mu_m)/[\mu_m+\mu_m(9K_m+8\mu_m)/(6(K_m+2\mu_m))]},\\ \frac{K-K_m}{K_c-K_m}&=\frac{\upsilon_c}{1+\upsilon_m(K_c-K_m)/(K_m+4\mu_m/3)},\end{aligned}\tag{1}$$