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Application of Mori–Tanaka method in 3–1 porous piezoelectric medium of crystal class 6

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We consider a piezoelectric medium of hexagonal crystal system of class 6 containing a uniform distribution of circular cylindrical inclusions aligned with the axis of material symmetry. We determine the piezoelectric Eshelby tensor explicitly. In the case of cylindrical holes, we use the Eshelby tensor together with the multiscale Mori–Tanaka Method to obtain analytical formulae for the effective electroelastic properties of the homogenized medium. These formulae depend upon both the electroelastic properties of the matrix material and the volume fraction of the cylindrical holes. Using electroelastic properties reported in the literature, we obtain graphs of the effective properties of the porous medium versus the volume fraction of the pores and show that these effective electroelastic properties decrease for increasing porous volume fraction, as expected. In particular, for the case of the crystal subclass 6 mm, the results obtained via Mori–Tanaka Method agree well with results obtained via Method of Asymptotic Homogenization and Finite Element Method. This work is important in the evaluation of the effective electroelastic properties of heterogeneous solids with hierarchical structures, such as bones.

1. Introduction

The focus of this research is on the evaluation of effective properties of heterogeneous piezoelectric materials. Different homogenization schemes, such as the Mori–Tanaka method (Benveniste, 1987; Mori & Tanaka, 1973) and the Asymptotic Homogenization method (Bravo-Castillero, Guinovart-Diaz, Sabina, & Rodríguez-Ramos, 2001), have been applied in the evaluation of these effective properties. These materials exhibit electromechanical coupling, which can be used to generate mechanical deformation when subjected to an electric field and become electrically polarized under mechanical loads. The piezoelectric crystals are all dielectric (Fang & Liu, 2013). From the 32 known crystal classes (Fang & Liu, 2013; Nye, 1985), 20 classes can generate the piezoelectric effect and are given by 1, 2, m, 222, 2mm, 4, $\bar{4}$, 422, 4mm, $\bar{4}2$m, 3, 32, 3m, 6, $\bar{6}$, 622, 6mm, $\bar{6}$m2, 23, $\bar{4}3$m. In this work we consider a medium composed of materials belonging to the crystal class 6 and its subclasses 6 mm and 622.

This research is motivated by the need of evaluating the effective properties of structural levels in hierarchical structures. One example of composite material with hierarchical organization is bone. Modeling bone and evaluating its effective

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properties represent a current challenge, in part, because available data in the literature is scarce. Experimental observations indicate that bone as a whole and collagen in particular are piezoelectric materials possessing the general hexagonal symmetry of class 6 (Fukada & Yasuda, 1957; Fukada & Yasuda, 1964; Yoon & Katz, 1976). Measurements performed by Tofail et al. (2009) on macroscopic piezoelectricity in sintered hydroxyapatite (HA) indicate the piezoelectric nature of HA. In spite of the fact that in living, or, wet bone, piezoelectric effects seem to be very minor and dominated by electrokinetic effects, streaming currents and zeta potentials, recently, a new interest has appeared to show the importance of bone piezoelectricity in bone's response to loading (Ahn & Grodzinsky, 2009; Fernández, García-Aznar, & Martínez, 2012; Lemaire, Capiez-Lernout, Kaiser, Naili, & Sansalone, 2011). Since experimental measurements of the material properties of bone in all levels are difficult, a multiscale modeling of bone together with a homogenization method become very important to calculate its effective properties.

In this work we present a two-scale model of bone and use the Mori–Tanaka method to evaluate the effective electroelastic properties of cortical bone from its properties at the mesoscale. At the mesoscale (mesostructural level) the cortical bone contains randomly arranged osteons embedded in the interstitial lamella, with some resorption cavities. The osteon is a long narrow cylinder consisting of concentric layers (rings) of lamellae, oriented in different directions, surrounding a long hollow Haversian canal (Hamed, Lee, & Jasiuk, 2010). Thus, at the mesoscale, dry cortical bone can be modeled as a porous medium containing unidirectional cylindrical holes embedded in a matrix made of piezoelectric material of crystal class 6 (Fukada & Yasuda, 1957; Fukada & Yasuda, 1964).

The Eshelby tensor is a key to the determination of effective properties of piezoelectric composites using the Mori–Tanaka method. Eshelby (1957) has shown that the problem of inclusions embedded in an infinite elastic matrix and subjected to a uniform eigenstrain can be solved elegantly by using the superposition principle of linear elasticity together with the Green's function. Deeg (1980) generalized the Eshelby's equivalent inclusion approach to obtain expressions for the coupled electroelastic fields in an ellipsoidal inclusion and inhomogeneity in an infinite piezoelectric matrix. Also, in that work the coupled electroelastic solutions were expressed in terms of electroelastic Green's functions. Benveniste (1992) and Dunn and Taya (1993a) have given a similar treatment for the ellipsoidal inclusion problem. Benveniste (1992) has obtained the coupled electroelastic fields for a single ellipsoidal inclusion problem without giving estimations of averaging fields. Based upon the Green's functions derived by Deeg (1980) and generalizing the effective medium theory of Mori–Tanaka to piezoelectric composites, Dunn and Taya (1993a) have used the solution for the single piezoelectric inhomogeneity to obtain average electroelastic fields at finite concentrations. A significant contribution to solve the equivalent Eshelby inclusion problem in a piezoelectric medium was given by Dunn and Taya (1993b,1993c). These authors have solved the equivalent Eshelby inclusion problem for a single ellipsoidal inclusion in an infinite piezoelectric medium applying the Green's function approach and the Mori–Tanaka method. The schemes to derive the electroelastic Eshelby tensor, as well as the explicit results for the tensor components of fibrous and lamellar composites, were also given by Dunn (1994), Huang and Yu (1994), Mikata (2000), Mikata (2001), and, more recently, by Wang and Weng (2016), Wang, Su, Li, and Weng (2015).

In this work we derive explicit expressions for the components of the Eshelby tensor for a piezoelectric material of class 6. To the best of our knowledge, the only known results for the components of the Eshelby tensor are those for particular cases of hexagonal crystals of class 6 mm (Benveniste, 1992; Bravo-Castillero et al., 2009; Dunn, 1994; Dunn & Taya, 1993a, 1993b, 1993c; Huang & Yu, 1994; Mikata, 2000; Mikata, 2001) and of class 622 (Aguiar, Bravo-Castillero, Rodríguez-Ramos, & Silva, 2013). Similar concepts used by Dunn and Taya (1993c) are used here to evaluate the influence pores on the resulting electroelastic behavior. We show explicitly the Eshelby tensor for the case of cylindrical inclusions aligned with the axis of material symmetry, which is parallel to the x_3-axis. When the piezoelectric material is of subclass 6 mm the components of the Eshelby tensor are equivalent to those obtained by Dunn and Taya (1993b, 1993c). Here, we present a direct and easier way to derive the integral representations of these tensors.

We would like to point out that we could have used the closed-form expressions for the Green's functions presented by Berndt and Sevostianov (2016) to obtain explicit expressions for the components of the Eshelby tensor. This approach is detailed in Dunn and Wienecke (1997). Instead, we have followed the approach used by Dunn (1994). Even though this author obtains explicit formulae for the coupled electroelastic Green's functions of a transversely isotropic piezoelectric solid in the first part of his work, he uses an expression similar to (25) below to relate the second derivatives of the Green's functions to the components of the inverse of a matrix with components given by (21) also below. In the case of circular cylindrical inclusions, the components of this matrix are easily obtained analytically and yield the components of the Eshelby tensor.

In summary, we obtain easy-to-implement analytical formulae for the effective electroelastic properties of a solid made of a piezoelectric material and containing a distribution of empty cylindrical holes with circular cross sections. For this, we first evaluate the Eshelby tensor for an infinite medium containing an ellipsoidal inclusion, both of which are piezoelectric of class 6. Next, we consider the limit case of a cylindrical inclusion with circular cross section and apply the Mori–Tanaka method in the analysis of a two-phase composite for which the matrix material is the same material of the infinite medium and the inclusions become empty cylindrical holes.

The analytical formulae depend upon both the electroelastic properties of the matrix material and the volume fraction of the cylindrical holes. These formulae are used to obtain numerical results for different case studies. For the case of a matrix with electroelastic properties reported in Kar-Gupta and Venkatesh (2006), graphs of the resulting effective properties versus the volume fraction of the pores are obtained. It is shown that these effective properties decrease for increasing porous