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The modeling of electrical property in porous media based on fractal leaf vein network

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ABSTRACT

The fractal leaf vein model has been proposed to investigate the electrical property of porous media in this work. The characteristics and evolution of fractal leaf vein network are described in detail. Besides, the structural parameters and corresponding electrical resistance of fractal leaf vein model are addressed by analytical solutions. The influences of structural parameters, like diameter ratio, length ratio, level number and so on, on electrical resistance of fractal network have been discussed profoundly. What is more, the comprehensive effects of temperature, concentration of electrolyte solution and applied frequency on electrical conductivity of pore solution have been significantly analyzed in this work. The interactions among kinds of structure parameters of fractal leaf vein model are studied at last.

1. Introduction

Porous media, such as cement paste (Tang et al., 2016; Tang, Chen, Shao, & Li, 2015; Tang, Li, Zhu, Shao, & Chen, 2014), concrete (Chen & Leung, 2014; Tang et al., 2016; Tang, Yao, Andrade, & Li, 2015), biological tissue with vascular network (Li, Liang, Yu, & Yang, 2014), rock and soil (Ding, Shi, Luo, & Wang, 2016), exist extensively in nature. There are a great number of physical properties of porous media, just like mechanical strength (Johansson, Wikberg, Ek, & Alderborn, 1995), permeability (Ortiz-Landeros, Norton, & Lin, 2013), heat conductance (Lu, Caps, Fricke, Alviso, & Pekala, 1995) and electrical transportation (Nguyen, Pham, Vu, & To, 2016; Wright, Cashman, Gottesfeld, & Roberts, 2009), related to intrinsic pore structure closely.

As a matter of fact, electrical property, as a significant performance of porous media, can influence their plenty of properties intensively. For example, electrical properties of porous electrodes, play profound role in energy transfer, which may provide a potential for sustainable development in energy field (Li, Liu, & Zhao, 2016). In addition, on the pore morphology and distribution aspect, pore structure at a certain range of scales in the porous media presents fractal characteristics (Diamond, 1999; Winslow, Bukowski, & Young, 1995), and these fractal features may be theoretically described by some fractal models, just like one dimensional Koch curve (Papadopoulos, Bird, Mooney, & Whitmore, 2008), two dimensional Sierpinski carpet (Tarafdar, Franz, Schulzky, & Hoffmann, 2001), three dimensional Menger sponge (Sergeyev, 2009; Tang et al., 2017; Vita, De Bartolo, Fallico, & Veltri, 2012), intermingled fractal units model (Pia, Sassoni, Franzoni, & Sanna, 2014) and fractal network (Miao, Yu, Duan, & Fang, 2015; Wang & Yu, 2011; Xu, Yu, Yun, & Zou, 2006; Zheng, Xu, Yang, & Yu, 2013). Fractal tree-like branched network was utilized by Xu et al. in 2006 to study the influence of geometrical parameters on the

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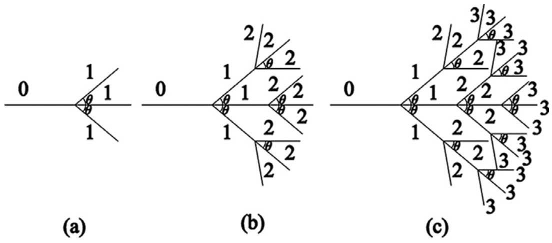

Fig. 1. Schematic illustration of fractal leaf vein network at first (a) two, (b) three, and (c) four levels.

effective thermal conductivity of network (Xu et al., 2006), but they did not consider the role of diameter and length of pore channels. Based on Sierpinski carpet fractal model, Pia and Sanna in 2014 studied the influence of pore size distribution of porous media on thermal conductivity and found that the presence of pore wall and small pores could eliminate thermal conductivity (Pia & Sanna, 2014). Nevertheless, some pore geometrical parameters (pore diameters, pore lengths and angle between adjacent pore channels) cannot be shown visually in the Sierpinski carpet. Tang et al. proposed a two-dimensional fractal-like model to represent the pore structure of cement pastes in 2015, and utilized this model to study the thermal conductance of cement pastes. They claimed the small value of porosity and iteration number or large value of minimal pore length of fractal-like model were beneficial to increase thermal conductivity of cement pastes (Tang et al., 2015). However, the branching number of model presented by Tang et al. was so limited that the model cannot represent the real porous media effectively. Wei et al. in 2015 deduced a relationship expression among electrical conductivity, porosity, and pore and tortuosity fractal dimensions to investigate the electrical property of fractal porous media. Besides, they analyzed correlation expressions of various fractal dimensions, including random walker, pore and tortuosity fractal dimensions (Wei et al., 2015). However, they did not take the impacts of temperatures and concentrations of electrolyte solution and applied frequencies on electrical conductance of porous media into account.

In this work, the fractal leaf vein network, is constructed to simulate and analyze the pore structure in porous media. It makes an attempt to characterize the pore fractal features and reveal the mechanism of electrical properties of porous media. Moreover, mutual effects among different structure parameters of fractal leaf vein model are addressed in the end.

2. Characteristics of the fractal leaf vein network

The proposed fractal leaf vein network that is inspired by the leaf vein in nature is adopted to simulate the pore structure network in porous media. In this work, the branch number is assumed to be 3 for the purpose of simplicity. The schematic illustrations of fractal leaf vein network at first two, three and four levels are shown in Fig. 1. Actually, the level number, n, of fractal leaf vein network can go to infinite. The evolution process of fractal leaf vein network can be summarized as:

(1) Mother channel at Level 0, has three identical son channels at Level 1, as shown in Fig. 1(a), and the middle son channel is located in the extension cord of mother channel. The angle θ between two adjacent channels at the same level is constant for the whole fractal network.
(2) Three son channels at Level 1 respectively develop three identical next generation son channels at Level 2, and the evolution rule is same as Step (1), as shown in Fig. 1(b).
(3) Each channel of three-channel cluster in main stem at Level 2 evolves three identical next generation three-channel clusters at Level 3, while only the middle channel of three-channel clusters located in two laterals of main stem at Level 2 can evolve next generation three-channel cluster at Level 3, as shown in Fig. 1(c).
(4) The generated channels repeat Step (3) to Level n, and then, the whole fractal leaf vein network is established.

For the sake of depicting the fractal leaf vein network advantageously, two assumptions are made in this work: (1) each channel in the network is straight; (2) channels do not overlap each other.

The mother channel at Level 0 has diameter d_0 and length l_0. Similarly, d_k and l_k are the diameter and length of channels at Level k, respectively. Besides, two scale factors, diameter ratio α and length ratio β of pore channels, are defined to correlate the size of channels at adjacent levels, as demonstrated by Eqs. (1)–(4):

$$\alpha = d_k/d_{k-1} \tag{1}$$

$$\beta = l_k/l_{k-1} \tag{2}$$

$$d_k = d_0\alpha^k \tag{3}$$

$$l_k = l_0\beta^k \tag{4}$$