International Journal of Engineering Science 123 (2018) 236–289

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Indentation on a transversely isotropic half-space of multiferroic composite medium with a circular contact region

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ARTICLE INFO

Article history:
Received 12 June 2017
Revised 9 November 2017
Accepted 15 November 2017

Keywords:
Multiferroic composite media
Contact problem
Circular contact region
Fundamental solution
Continuous profile
Concave indenter

ABSTRACT

The present paper concerns the contact problem in the context of elasticity of multiferroic composite media. The substrate under consideration is a half-space which is made of magneto-electro-elastic material with transverse isotropy. The magneto-electric properties of the indenter are comprehensively taken into account. The integral equations for four cases are established by means of the general solution and the generalized potential theory method. The corresponding fundamental magneto-electro-elastic field variables in the substrate, which are caused by the generalized indentation displacements following the Dirac-delta distribution in the contact area, are explicitly and exactly obtained in terms of elementary functions. Based on the present fundamental solutions, a general theory of indentation, where the profile of the indenter may be expanded as a Taylor series, is developed. Furthermore, by applying the results of a general theory, two classical concave indenters (conical and parabolic) are considered. Concept of effective penetration depth is proposed for concave indenters. Numerical calculations are carried out in order for various purposes, including validation of the present solutions, demonstration of the coupling effects in multi-phases of the composite medium and effect of the geometry of the indenter on contact behaviors.

1. Introduction

With the development of science and technology, a growing list of non-traditional materials with multi-field coupling effects, such as piezo-electric and piezo-magnetic media, are used in numerous industrial fields, to cater for the requirements in the era of intelligent materials (Gandhi & Thompson, 1992). Magneto-electro-elastic materials or multiferroic composite media, in thin-film and bulk forms, may have great potential applications in various fields of innovative and high technology, such as high-density and high-performance memories in data storage engineering. The mechanical, electrical and magnetic behaviors of multiferroic composite media are of high significance in design and operation of intelligent systems. Their responses, such as the polarization direction, the polarization switching behaviors, losing in piezo-electric and piezo-magnetic effects, aging response and activation energy for depolarization, may be detected through a continuous record of

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https://doi.org/10.1016/j.ijengsci.2017.11.013
0020-7225/

the responses during instrumented indentation (Chen, Pan, Wang, & Zhang, 2010; Giannakopoulos & Parmaklis, 2007; Giannakopoulos & Suresh, 1999; Kalinin et al., 2007a). Multiferroic composite materials, which combine both piezo-electric and piezo-magnetic phases and exhibit stronger magneto-electric coupling effect in comparison to single piezo-electric and piezo-magnetic phases, have recently stimulated intensive academic and technological interest (Eerenstein, Mathur, & Scott, 2006; Zheng et al., 2004).

To characterize the mechanical and non-mechanical properties of these materials, techniques of scanning probe microscope and nanoindentation experiment came into advent in the past two decades (Giannakopoulos & Suresh, 1999; Kalinin, Karapetian, & Kachanov, 2004; Kalinin, Mirman, & Karapetian, 2007b; Kalinin et al., 2007a; Karapetian, Kachanov, & Kalinin, 2005, 2009). These powerful new techniques evidently depend upon the understanding of contact problems in the context of elasticity of multifield coupling materials (Chen et al., 2010; Giannakopoulos & Suresh, 1999; Karapetian, Kachanov, & Kalinin, 2009). The studies on contact problems in the framework of elasticity of multifield coupling materials, which are helpful to interpret quantitatively the response of the various scanning probe microscopies, may bridge the mechanician and material scientists (Chen, 2015).

To date, a great deal of scientific effort has been made on the theory of indentation for a piezo-electric or piezo-magnetic materials (Giannakopoulos, 2000; Giannakopoulos & Parmaklis, 2007; Giannakopoulos & Suresh, 1999; Kalinin et al., 2004; Kalinin et al., 2007b; Kalinin et al., 2007a; Wang, Chen, & Lu, 2008). A comprehensive literature review on this topic for piezo-electric and piezo-magnetic materials is far beyond the scope of the present paper. The interested reader may refer to the review articles (Chen & Ding, 2004; Chen et al., 2010; Kalinin et al., 2007b) and the recent studies (Chen et al., 2010; Karapetian et al., 2009; Li, Wu, Jin, & Chen, 2015).

From the beginning of this century, researchers began to pay attention to the contact problems for multiferroic materials. For instance, Hou, Leung, and Ding (2003) developed the three-dimensional exact solutions of the elliptical Hertz contact for two spheres of magneto-electro-elastic materials, which are subjected to a pair of concentrated forces and a pair of point electric charges. In this study, both smooth and frictional circumstances have been taken into consideration. Then, Chen et al. (2010), starting from the Green's functions of a half-space of multiferroic composite medium and generalizing the potential theory method (Fabrikant, 1989, 1991), studied the axisymmetric contact problems in a systematic way. A general theory of indentation over an infinite half-space of the transversely isotropic multiferroic composite medium, which are punched by three common indenters (namely, flat-ended, conical and spherical circular punches), was presented. Rogowski and Kalinski (2012) investigated the indentation of a magneto-electro-elastic half-space punched by a truncated conical indenter. For a half-space of multiferroic composite medium punched by a half-infinite indenter, fundamental magneto-electro-elastic field in terms of elementary functions was obtained by Li, Zheng, and Chen (2014). Inspired by Chen et al. (2010), Li et al. (2015) studied the elliptical contact problem, with the help of the recent progresses of potential theory method (Fabrikant, 2005; 2007).

As a smart material, the multiferroic composite medium is used in broad and important "contact prone" applications, where the indenters with various magneto-electric properties may be involved. Indenters are axisymmetric in most applications, and the researchers often focus on the three common indenters (Chen et al., 2010; Giannakopoulos & Suresh, 1999; Wang et al., 2008). However, the axisymmetric indenter in practice may have more complicated geometrical configurations, other than the foregoing three punches. Further, the contact stiffness extracted from the experimental data via atomic force acoustic microscopy is sensitive to the geometry of the indenter, as indicated by Karapetian et al. (2009). To cater for the engineering requirements, the indenters may have a variety of geometrical configurations. Hence the profiles of some axisymmetric indenters may be mathematically formulated by arbitrary continuous functions, including those pertinent to the flat-ended, conical and cylindrical punches (Dhaliwal & Rau, 1970). From the historical perspective, the pure elastic contact problems with an axisymmetric arbitrary profile were investigated sequentially by Sneddon (1965) and Dhaliwal and Rau (1970), making use of Hankel transform and iterative method of integral equation, respectively. Further, on the micro-scale, the penetration depths between the matrix and the indenter may vary in a discontinuous fashion, but follow a certain probability density function (*e.g.*, Gaussian distribution) in the statistical sense (Greenwood & Williamson, 1966).

In the context of magneto-electro-elasticity, Rogowski (2012) studied the indentation of a magneto-electro-elastic half-space or a layer on a two-parameter elastic foundation punched by a cylindrical indenter with a slightly concave base. The profile of the concave base is assumed to be conical or parabolic (see Fig. 3). The hypothesis that the indenter is both electrically and magnetically conducting was adopted by Rogowski (2012). For the half-space punched by a cylindrical indenter, the field variables were given in a closed form by virtue of the Hankel transform. The critical indentation force corresponding to the full contact condition was derived as well. When the indentation force is smaller than those thresholds corresponding to an annular contact region, the approximate solutions were proposed via the method of triple integral equations and series solution technique (Rogowski, 2012).

It should be pointed out that the expressions of the field variables developed by Rogowski (2012) involve several parameters, which turns out to be four sets of oblate spheroidal coordinates and are determined implicitly by two coupling equations (See Appendix 4 therein). In fact, the solutions to axisymmetric crack and contact problem derived by the Hankel transform generally invoke integrals of Bessel functions, which are generally difficult to evaluate explicitly. This difficulty may be overcome by means of the potential theory method initiated by Fabrikant (1989, 1991).

In the studies conducted by Hou et al. (2003) and Rogowski and Kalinski (2012), the indenters are assumed to be electrically and magnetically conducting. In recent investigations (Chen et al., 2010; Li et al., 2015; Li et al., 2014), however, the indenters may be magnetically and electrically insulating, and more complicated situations therefore have been taken