International Journal of Engineering Science 122 (2018) 30–55

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Hybrid energy transformation to generalized Reissner–Mindlin model for laminated composite shells

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ARTICLE INFO

Article history:
Received 18 July 2017
Revised 25 August 2017
Accepted 15 September 2017

Keywords:
Laminated composite shells
Variational asymptotic method
Hybrid energy transformation
Generalized Reissner-Mindlin model

ABSTRACT

By using the variational-asymptotic method, a two-dimensional mechanical model for laminated composite shells is established from a mathematical perspective, having the energy functional asymptotically correct up to the desired order in the small parameters. However, it is not in a practical form from an engineering perspective because of the appearance of partial derivative terms, which bring unnecessary mathematical complexity and obscure physical interpretation of mechanical boundary conditions in the shell modeling. Therefore, one more procedure is inevitably required – a so-called energy transformation procedure, which constructs a mathematical link between the energy functional derived herein and a simpler engineering model, such as a generalized Reissner–Mindlin model. In a different manner from previous works, this article introduces a hybrid energy transformation procedure composed of two successive steps: an equilibrium transformation via a linear algebraic approach, and an energy transformation via a perturbation approach. During this procedure, the first step is to transform the two-dimensional equilibrium equations from a hyper-static system into an isostatic system by augmenting them with the two-dimensional compatibility equations. Then, for obtaining a generalized Reissner–Mindlin model, the second step is to introduce initial curvature/twist as small parameters (in the sense of perturbations) into the constitutive law. The coupling stiffness terms between the transverse shear generalized strain measures and the remaining generalized Reissner–Mindlin strain measures are shown to be identically zero for laminated composite plates/shells (in contrast to the analogous terms of a similarly constructed generalized Timoshenko model for composite beams). Several examples are presented to demonstrate the capability and accuracy of this new approach.

1. Introduction

Laminated composites have received considerable attention for use in the automotive, aeronautical, and construction industries for lightweight structures with high strength, superior energy absorption capability, and/or high thermal resistance under a variety of extreme environments. Furthermore, along with increased fabrication knowledge and improved manufacturing techniques, they have demonstrated more potential to achieve the ever-increasing performance requirements by

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https://doi.org/10.1016/j.ijengsci.2017.09.006

designing new structures with optimized arrangements of different geometric and material properties. For those reasons, in order to estimate various mechanical behaviors of laminated composites, research on accurate and general modeling for them has consistently remained as a very active field in the last several decades. Obviously, a three-dimensional (3D) elasticity formulation is capable of modeling such structures and accurately calculating the details of their behavior to any degree of generality. To do so, however, can be quite complex and costly. Clearly, there is a need for alternative, less costly approaches than 3D elasticity.

From physical and geometrical perspectives, laminated composites with initial curvature/twist can be modeled as two-dimensional (2D) shells by taking advantage of the smallness of the thickness to the wavelength of deformation (h/l) and the thickness to radius of curvature/twist (h/R). As a result, researchers have strived to construct 2D shell models under various ad hoc kinematic assumptions and distinctive dimensional reduction processes, which are convenient to eliminate the through-thickness coordinate from the independent variables of the governing partial differential equations. Referring to the existing literature, one can easily observe that there has been a tremendous amount of work done to construct shell models, most of which differ from each other only by their specific ad hoc kinematic assumptions and/or because of their specific dimensional reduction process; see Cho and Averill (2000), Noor and Malik (2000), Reddy and Arciniega (2004), Carrera and Brischetto (2009), Demasi and Yu (2013) and Han and Bauchau (2016). Unfortunately, because most models, such as classical lamination theory (CLT) and first-order shear deformation theory (FOSDT), rely on ad hoc kinematic assumptions and attendant simplistic dimensional reduction procedures, they do not always perform well. Such plate/shell theories have always suffered from unbalanced compromise between efficiency and accuracy, or between simplicity and generality, for predicting mechanical behavior of curved/twisted shells made of laminated composites.

In recent decades, however, the variational-asymptotic method (VAM) introduced by Berdichevsky (1979) has been used to construct quite accurate and general models for laminated composite shells found in Yu, Hodges, and Volovoi (2002b). In this approach, the original 3D elasticity problem is first formulated in an intrinsic form suitable for the geometrically nonlinear analysis, which can accommodate arbitrarily large global displacement and rotation, subject only to the maximum strain being small. Then, considering the smallness of the two parameters (h/l and h/R), dimensional reduction using VAM rigorously splits the nonlinear 3D problem into a nonlinear 2D surface analysis and a linear one-dimensional (1D) through-the-thickness analysis. Here solution of the 1D through-the-thickness analysis provides a set of 2D elastic constants (i.e., a constitutive law) to be used in the 2D surface analysis, along with 3D recovery relations that yield the 3D displacement, strain, and stress fields using results calculated from the solution of the 2D surface analysis. In particular, the 1D through-the-thickness analysis is based on an energy functional that is asymptotically correct up through the first order of h/R and the second order of h/l in terms of the 2D shell variables and their partial derivatives.

A straightforward use of the asymptotically correct energy is possible in engineering applications. However, from an engineering perspective, such an approach is neither practical nor efficient because additional boundary conditions caused by the appearance of partial derivative terms are difficult to link with those engineering quantities normally specified on the boundary of shells. Therefore, to avoid this complexity, an energy transformation procedure seems to be required in order to form simple engineering models, such as the generalized Reissner–Mindlin (RM) model for plates/shells. In particular, for initially curved and twisted beams, research attention devoted to accurate and general energy transformation procedures into the generalized Timoshenko model has received considerable attention in the past decades; see Popescu and Hodges (2000), Yu (2002), Ho, Yu, and Hodges (2010), Yu, Hodges, and Ho (2012), Rajagopal, Hodges, and Yu (2012) and Rajagopal and Hodges (2012). However, there has been very little such work done for plates/shells. Only three procedures for transformation of the asymptotically correct energy into a "Reissner-like" model have been suggested in the literature, to the authors' knowledge. All of them strive to maintain the reduced strain energy to be as close as possible to being asymptotically correct. One of those, which is also referred to as a smart minimization procedure, was first carried out for laminated composite plates by Sutyrin and Hodges (1996) and Sutyrin (1997) without a known mathematical basis to produce the good agreement obtained. On the other hand, by relaxing the established warping constraints, a procedure based on optimization, together with a semi-definite programming technique (Toh, Todd, & Tutuncu, 1999), was proposed by Yu, Hodges, and Volovoi (2002a) and Yu (2005) to obtain a "Reissner-like" plate model, later generalized to handle laminated composite shells found in Yu et al. (2002b). A somewhat different "hybrid" approach was recently introduced by Lee and Hodges (2015). It involves modifying the 2D intrinsic equilibrium and compatibility equations and solving a system of resulting linear algebraic equations via the pseudo-inverse method. Although relaxation of the well-established warping constraints and introduction of complex optimization and smart minimization procedure were not required in this approach, two important inconsistencies inherent in the energy transformation procedure were still not resolved in Lee and Hodges (2015): (1) the work did not provide a general procedure for transformation from the hyper-static system of static equilibrium equations into an isostatic system, and (2) because the work was only focused on plates, it did not present a systematic path to extend the plate analysis to shells. (According to Bauchau and Craig (2009), the term "hyper-static" means that the number of force unknowns is *larger than* the number of equilibrium equations, while the term "isostatic" means that the number of force unknowns is *the same as* the number of equilibrium equations.)

The main objective of this paper is to resolve these two inconsistencies by performing a hybrid energy transformation, which is composed of two successive steps: (1) a transformation of the 2D static equilibrium equations via a linear algebraic approach and (2) an energy transformation via a perturbation approach in terms of small parameters. By performing a hybrid energy transformation procedure, the transformation from a hyper-static system of 2D equilibrium equations into an isostatic system is first carried out by taking 2D compatibility equations into account simultaneously. For the systematic