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Nonlinear oscillations of functionally graded microplates

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ABSTRACT

The size-dependent nonlinear oscillation characteristics of a functionally graded microplate is investigated numerically, in which all the displacements, i.e. in-plane as well as out-of-plane, and their inertia are accounted for. The potential energy of the functionally graded microsystem is obtained based on a modified version of the couple stress theory, so as to account for size effects, together with the Mori-Tanaka homogenisation mixture model for the graded material property. The kinetic and size-dependent potential energies of the microsystem are dynamically balanced by the work of an external force via the Lagrange equations and truncated employing an assumed-mode discretization scheme. Extensive numerical simulations are conducted upon the discretised model of the microsystem through use of a continuation technique as well as an eigenvalue extraction method (for the nonlinear and linear studies, respectively). The effect of several functionally graded microsystem parameters, namely the material gradient index, the material length-scale parameter, and the frequency and amplitude of an exciting external force on the response is investigated.

1. Introduction

1.1. Applications and fundamentals

Many microelectromechanical applications, ranging from micro sensors to energy-harvesting microsystems, have motivated a considerable amount of research on the vibrations of micro continuous elements such as microplates and microbeams (Asghari, Kahrobaiyan, & Ahmadian, 2010; Baghani, 2012; Ghayesh & Farokhi, 2015a, Ghayesh, Farokhi, & Amabili, 2013, Zhang & Meng, 2007; Zheng, Dong, Lee, & Gao, 2009). Demands for designing microelectromechanical systems with lighter materials, novel configurations, and varying thickness, in order to achieve better performance and sensitivity, are the reasons for the birth of microelements made of functionally graded materials (FGM) (Huang, 2008; Lü, Lim, & Chen, 2009; Witvrouw & Mehta, 2005) . Resistance to high temperature due to the ceramic-rich face and high structural strength owing to the metal-rich face is the two-fold advantage of these classes of composites in microelectromechanical applications.

Conducting an investigation into the vibration behaviour of functionally graded microplates helps us understand the resonant motion characteristics of the more complex real-world microelectromechanical systems, and hence enhance their performance. The highly size-dependent behaviour of microelements which was observed experimentally in Chasiotis and Knauss, (2003), Lam et al. (2003), Liu et al. (2012) and Nix (1989) shows that small size can dramatically alter the

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resonance behaviour; thereby, employing a higher-order continuum mechanics theory, such as the modified version of the couple stress based strain gradient theory (Dai, Wang, & Wang, 2015; Dehrouyeh-Semnani, Dehrouyeh, Torabi-Kafshgari, & Nikkhah-Bahrami, 2015; Farokhi & Ghayesh, 2015a, b, Farokhi, Ghayesh, & Amabili, 2013; Ghayesh & Amabili, 2014; Ghayesh & Farokhi, 2015b, Ghayesh, Farokhi, & Amabili, 2013, Ghayesh, Farokhi, & Amabili, 2014; Ghayesh, Farokhi, & Hussain, 2016; Gholipour et al., 2015; Li & Pan, 2015, Şimşek, 2010; Tang, Ni, Wang, Luo, & Wang, 2014) is essential to eliminate the gap between experimental observation and classical continuum mechanics predictions (Akgöz & Civalek, 2013; Akgöz & Civalek, 2011; Dehrouyeh-Semnani, 2014; Dehrouyeh-Semnani, BehboodiJouybari, & Dehrouyeh, 2016; Farokhi, Ghayesh, Gholipour, & Hussain, 2017; Ghayesh, Amabili, & Farokhi, 2013a, Hosseini & Bahaadini, 2016; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Karparvarfard, Asghari, & Vatankhah, 2015; Kong, Zhou, Nie, & Wang, 2008; Mojahedi & Rahaeifard, 2016; Shafiei, Kazemi, & Ghadiri, 2016a, b, Taati, 2016).

1.2. Literature review

The literature regarding the size-dependent vibration/bending/buckling of rectangular functionally graded microplates can be divided mainly into two general classes in terms of the employed models and assumptions; these two classes are linear and nonlinear. Starting with the review of the *linear* analyses, for instance, Ansari, Gholami, Faghih Shojaei, Mohammadi, and Darabi (2013) contributed to this field by studying the buckling behaviour of a Mindlin functionally graded rectangular microplate subjected to a thermal loading; a strain gradient theory was employed to account for size effects. Nami and Janghorban (2014) analysed the deflection ratio of an FGM rectangular microplate by analysing the linear natural frequencies and resonant responses. Lei et al. (2015) employed a higher-order theory and the modified couple stress based theory so as to examine the linear natural frequencies and shear stresses of an FGM microplate. Zhang, He, Liu, Shen, and Lei (2015) analysed the bending and buckling as well as free vibrations of a functionally graded microplate on an elastic foundation via use of a strain gradient elasticity theory. Li and Pan (2015) conducted an investigated on the bending behaviour as well as free oscillations of an FGM piezoelectric microplate by means of a size-dependent continuum theory. A closed-form analytical solution for the buckling behaviour, as well as post-buckling response, of a functionally graded microplate was derived by Taati (2016) employing a size-dependent couple stress based theory.

The literature on the second class which analysed the *nonlinear* vibration/bending/buckling of functionally graded rectangular microplates is *not* large. Jung, Park, and Han (2014) developed a nonlinear model of an FGM microplate embedded on an elastic medium by means of the modified couple stress theory, then conducted a linear analyses on the bending behaviour and natural frequencies. Ansari, Faghih Shojaei, Mohammadi, Gholami, and Darabi (2014) contributed to the field by analysing the nonlinear *free* vibrations of a functionally graded Mindlin microplate via use of the modified couple stress theory where no stability analysis was provided. Lou and He (2015) examined the *free* vibrations and bending behaviour of a functionally graded microplate by developing closed-form solutions employing the modified couple stress theory. He, Lou, Zhang, Wang, and Bai (2015) employed the modified couple stress theory in order to develop a four-variable model of FGM microplates so as to determine linear natural frequencies and bending behaviour. Lou, He, and Du (2015) developed a nonlinear model of a functionally graded microplate and then analysed the static buckling and bending behaviours as well as linear natural frequencies of the functionally graded microsystem.

1.3. Contributions of the present study

This study, for the first time, conducts an investigation into the *size-dependent nonlinear forced vibrations and stability* of a rectangular functionally graded microplate taking into account all the in-plane and out-of-plane displacements and inertia in the modelling and simulations. In this study, the microplate is modelled based on the modified couple stress theory along with the Mori-Tanaka homogenisation model in order to capture both size-dependent and non-homogenous characteristics, respectively. For the first time, all the displacement components, i.e. in-plane and out-of-plane ones, as well as their inertia are retained in the nonlinear model of a functionally graded microplate. To this end, the size-dependent potential energy, as well as the kinetic energy, of the functionally graded microsystem are balanced by the work of an external dynamic force via Lagrange equations; the resultant discretised model is then recast into a new set, consisting of first-order nonlinear ordinary differential equations (ODEs). The final set of equations is solved via use of a well-optimised pseudo-arclength continuation technique for the nonlinear response and the eigenvalue analysis for the linear response of the functionally graded microsystem. The effect of the length-scale parameter as well as the material gradient index on the nonlinear responses of the functionally graded microsystem is investigated thoroughly via building force-response and frequency-response curves.

2. Model development for functionally graded microplate

The functionally graded microsystem considered in this paper consist of a flexible rectangular microplate hinged at all edges which are not movable in the in-plane directions, fixed in a rectangular Cartesian coordinate system (x, y, z), as represented in Fig. 1. Microplate's thickness is h; the surface at $z=h/2$ (i.e. the upper surface) is made out of ceramic-rich and the one at $z=-h/2$ is metal-rich. The in-plane dimensions are denoted by a and b which are in x and y directions, respectively. The microplate is assumed to be subjected to a harmonic distributed force per unit area, $f_1\cos(\omega t)$, in the z