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Transmission and localisation in ordered and randomly-perturbed structured flexural systems☆

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ABSTRACT

The paper presents a novel analysis of localisation and transmission properties of randomly-perturbed flexural systems. Attention is given to the study of propagation regimes and the connection with transmission resonances following perturbations of ordered stacks. The analytical study is complemented with numerical simulations relevant to different discrete systems. Applications are in the design of efficient vibration isolation systems and filters of elastic waves considered as ordered structures subjected to small random perturbations.

1. Introduction

1.1. Periodic versus perturbed structures

Many structures are designed as assemblies of identical units, connected to each other with different kinds of joints. Such structures behave as filtering systems, since in some ranges of frequencies waves propagate without attenuation (if damping is neglected), while in others waves decay exponentially. For an infinite periodic system, these frequency ranges are denoted as *pass-bands* and *stop-bands*, respectively. A finite system exhibits similar transmission properties, provided that the number of its components is large enough. For a finite system, we distinguish between *propagation ranges* and *non-propagation ranges* since, strictly speaking, the definitions of pass-bands and stop-bands cannot be adopted.

Real structures are never perfect, as errors in manufacturing processes are likely to occur. The presence of defects and imperfections in the geometric and constitutive properties of the structure is generally referred to as "disorder". We focus the attention on elastic structures which are periodic prior to a perturbation. For such periodic structures Floquet–Bloch waves can be analysed in detail, but the approach has to change as perturbation is introduced. Disordered systems consisting of many units can be found at different scales, from photonic crystals at the nanoscale to mechanical and civil engineering structures at the macroscale. In this paper, we consider elastic media that can be modelled as discrete systems. For instance, arrays of tanks (Fig. 1a) can be studied as chains of masses connected by flexural links, which simulate the elastic plate supporting the tanks. In this case, disorder

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Fig. 1. Two engineering examples of perturbed (but otherwise ordered) structures: (a) oil storage tanks in a power generation plant (from http://www.chromalox.com, accessed on 30 July 2015); (b) failure of the San Saba bridge in Texas, occurred in May 2013 due to fire (image captured on 30 July 2015 from the video https://www.youtube.com/watch?v=LLVKb1HxhAY). In (a) the fluid level in tanks is variable and hence the inertia properties are perturbed; in (b) the elastic stiffness of pillars is perturbed along the length of the bridge.

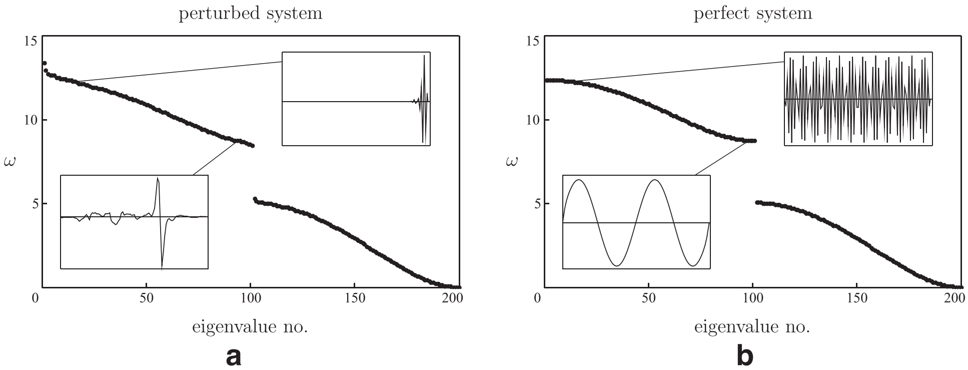

Fig. 2. Eigenfrequencies ω and examples of eigenvectors of a finite discrete flexural system, consisting of disks connected by massless beams, in the case of perturbed conditions (a) and in the perfectly ordered configuration (b).

can be represented by the randomly-varying amount of fluid inside the containers. Bridges can be studied as discrete sets of masses connected by non-inertial beams and resting on elastic supports, as in the analytical model developed by Brun, Giaccu, Movchan, and Slepyan (2014) to describe the recent collapse of the San Saba bridge in Texas (Fig. 1b). In the case of bridges, the random parameters could be the span lengths or the pillar heights.

Systems made of modular units can be classified according to the number of kinematic variables p defining the coupling between two adjacent units: *mono-coupled* if $p = 1$, *bi-coupled* if $p = 2$, and so on. The dynamic properties of mono-coupled disordered systems have been extensively investigated in the literature. Asatryan et al. (2010, 2012); Cetinkaya (1999); Chen and Wang (2007); Guenneau, Movchan, Movchan, and Trebicki (2008); Li, Wang, Hu, and Huang (2006); Sigalas and Soukoulis (1995) studied propagation of elastic waves with different angles of incidence in disordered layered structures, where the quantity undergoing a random perturbation is the thickness of the layers, the elastic constant of one phase or the sequence of the layers. The effect of disorder in continuous one-dimensional randomly-perturbed systems is described by Godin (2005) with the analysis of the Lyapunov exponent for exponentially decaying perturbations, and by Godin, Molchanov, and Vainberg (2011) by examining the dependence of the Lyapunov exponent on the frequency and on the magnitude of disorder. A simple mass-spring model is employed by Yan, Zhang, and Wang (2009) to analyse localised modes in a layered structure after introducing different sources of disorder. In the context of bi-coupled systems, Ariaratnam and Xie (1995); Bouzit and Pierre (2000); Li, Wang, Hu, and Huang (2004) investigated different aspects of wave localisation in continuous beams, resting on many supports and having random span length, using the notion of *localisation factor*, which is related to the Lyapunov exponents of the dynamic system. For a general discussion on localisation phenomena in engineering structures, the reader is referred to the review by Bendiksen (2000).

In this paper, we describe spectral and transmission properties of discrete flexural systems with random parameters, and we make a comparison with the properties of regular unperturbed systems. In particular, we discuss localisation of eigenvectors and eigenvalues of matrices with random entries, derived from the equations of motion of the disordered system, in the framework of spectral localisation for perturbed (but otherwise ordered) discrete elastic structures. We show that in a perturbed system localised resonance modes appear (Fig. 2a), while in a perfect system the eigenstates are spread over the entire structure (Fig. 2b). Isolated eigenvalues are detected in the spectrum of the perturbed system, especially near its boundaries (Fig. 2a). We demonstrate that there is a link between the eigenvalue problem and the energy transmission problem, since at the frequencies where