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On the isotropic and anisotropic viscosity of suspensions containing particles of diverse shapes and orientations

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ABSTRACT

The classical problem of effective viscosity of a Newtonian fluid containing rigid particles is discussed. For spherical particles, it is shown that the usual Einstein's formula $\mu/\mu_0 = 1 + 2.5\phi$ represents an incorrect formulation of the non-interaction approximation (NIA): it violates a rigorous lower bound for the effective viscosity. The correct formulation yields the effective viscosity in the form $\mu/\mu_0 = (1 - 2.5\phi)^{-1}$ that agrees with the bounds and remains accurate at substantial volume fractions of particles ϕ (up to 20%–30% according to various data sets). This result is extended to ellipsoidal particles, with the emphasis on *mixtures* of particles of diverse aspect ratios and cases of *anisotropic* viscosity (due to non-random orientations of particles). For mixtures of particles of diverse shapes (such as ellipsoids of diverse aspect ratios), the effective viscosity cannot generally be expressed in terms of either ϕ or any other of concentration parameter, and the very concept of concentration parameters becomes questionable. The case of thin platelets is considered in detail; in this case, the concentration parameter is identified, and it is different from the volume fraction.

1. Introduction

We consider the effective viscosity of a fluid containing rigid particles, in the limit of low Reynolds numbers (creeping flow). This is a classical homogenization problem where the key result belongs to Einstein (1906, 1911) who considered particles of the spherical shape in the non-interaction approximation. Using the solution for a flow around a spherical particle, he obtained the effective viscosity μ in the form

$$\mu/\mu_0 = 1 + 5/2\phi, \tag{1.1}$$

where μ_0 is the viscosity in absence of particles and ϕ is particles' volume fraction.

We point out two shortcomings of this result:

- Beyond correctly predicting the initial slope $(d\mu/d\phi)_{\phi=0}$, its agreement with experimental data is limited to relatively small values of ϕ;
- It violates a rigorous lower bound for the effective viscosity, at any value of ϕ.

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As discussed in the text to follow, the root of these shortcomings is that the non-interaction approximation (NIA) has two dual formulations that correspond to summation of either viscosity- or fluidity contributions of particles. The two coincide asymptotically, at $\phi \to 0$ [leading to a frequent confusion – identification of the NIA with the "dilute limit", or the "linear limit"; see, for example, books and reviews of Mewis and Wagner (2012), Stickel and Powell (2005), and Brader (2010)]. However, one of them violates the lower bound for the effective viscosity and has poorer agreement with data.

The present work addresses the following issues:

(A) *Spherical* particles: Proper formulation of the non-interaction approximation that leads to modification of the Einstein's Eq. (1.1);
(B) *Non-spherical* particles. For these cases, *explicit algebraic* extensions of the Eq. (1.1) – i.e. finding the coefficient at ϕ – has not been given in literature, to our knowledge. We mention the result of Jeffery (1922) concerning spheroidal particles (of identical aspect ratios) that provides two estimates of these coefficients corresponding to the minimal and maximal viscous dissipation. The Jeffery's result is actually obtained in the non-interaction approximation, for which the effective viscosities in the case of ellipsoidal particles can be obtained explicitly for an arbitrary orientation distribution, as shown in this work. (Note that Jeffery's estimates are obtained under the assumption that the viscosity is isotropic whereas the suspension considered by him is actually anisotropic: ellipsoidal particles tend to align themselves with the direction of the flow). Importantly, our analysis covers *mixtures* of diverse shapes – the problem of obvious practical relevance (e.g. Fig. 1);
(C) *Anisotropic* viscosity caused by non-random orientations of non-spherical particles, the background fluid assumed isotropic. Although anisotropy fluids have been considered in the literature [see e.g. Rajagopal (2006) and Perlacova and Průša (2015)], the anisotropic suspension does not seem to have attracted much attention in literature, it is not uncommon. Firstly, we mention that ellipsoidal particles gradually align themselves with the direction of the flow, leading to gradual development of anisotropy – the phenomenon that was hypothesized by Jeffery and experimentally observed (Saffman, 1956; Taylor, 1923; Mueller, Llewellin, & Mader, 2010). Further evidence of anisotropy is provided by Fig. 2 that shows red blood cells in a blood flow, in a random orientation state (Fig. 2A) and in nearly perfectly aligned state (2B). Non-random orientations have also been observed for polarized particles in electro-rheological fluids (Halsey, 1992) and in magnetized suspensions in magmatic flows (Mueller, Llewellin, & Mader, 2011a, 2011b).

Remark. The classical linear viscosity law is assumed in the present work. This excludes fluids exhibiting non-linear behavior (Rajagopal, 2006). Also, various complicating factors are ignored. In particular, we ignore the Brownian motion effects characterized by the Péclet number, $Pe = \mu_0 a^3 |e_{ij}|/(k_B T)$ where a is a characteristic size of the particle, $|e_{ij}|$ is the magnitude (Euclidean norm) of the symmetric part of the velocity gradient tensor, k_B is the Boltzmann constant and T is the absolute temperature. At $Pe >> 1$, macroscopic hydrodynamic forces are much larger than the ones related to the thermal motion of a particle, and this is the case assumed in the present analysis (for analysis of thermal motion of spheroidal particles we refer to works of Brenner (1972, 1974)).

As far as the issue (A) is concerned, extensive experimental data on suspensions containing spherical particles consistently show that the initial slope $(d\mu/d\phi)_{\phi=0}$ generally agrees well with Eq. (1.1); however, this equation quickly loses accuracy as volume fraction increases: the viscosity increases substantially faster than predicted (Ford, 1960; Mewis & Wagner, 2012; Oliver & Ward, 1953; Russel, Saville, & Schowalter, 1989). Computational simulations have led to similar conclusions (Chang & Powell, 1994a, 1994b; Phung, 1993).

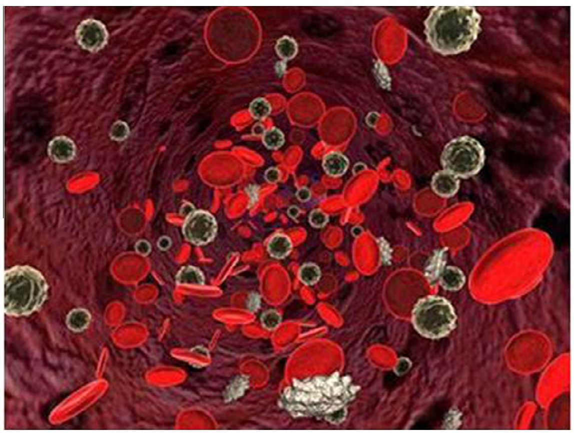

Fig. 1. Mixture of red blood cells, white blood cells, platelets and enzymes in the arteries. (Buzzle.com).