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Nonlinear dynamics of cantilevered microbeams based on modified couple stress theory

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ABSTRACT

The aim of this paper is to develop a new nonlinear theoretical model for cantilevered microbeams and to explore the nonlinear dynamics based on the modified couple stress theory, taking into account of one single material length scale parameter. The full nonlinear equation of motion, which is valid when the motion is large, is derived using the Hamilton's principle. The governing partial differential equation is further discretized with the aid of Galerkin's method. The numerical results, in which the existence of primary resonances of the first mode of the microbeam due to base excitations is demonstrated, are presented in the form of frequency–response curves, phase portraits and time histories. For a cantilevered microbeam subjected to harmonic base excitations, it is found that the frequency–response curve exhibits a clear softening-type behavior. For the same system but with an intermediate linear spring support, it is shown that the linear spring is capable of increasing the resonance frequency and decreasing the resonance amplitudes of the microbeam. Interestingly, it is found that the softening behavior could be changed to a hardening one if an intermediate nonlinear spring is added somewhere along the microbeam's length.

1. Introduction

Microbeams have become one of the core structures widely used in the field of micro-electro-mechanical systems (MEMS) (Kong, Zhou, Nie, & Wang, 2008). In the literature, it was reported that a large number of potential applications utilized the dynamical behaviors of microbeams for targeted performance specifications such as those vibration shock sensors (Lun, Zhang, Gao, & Jia, 2006), atomic force microscopes (Wang & Hu, 2005), accelerators, biosensors (Farokhi & Ghayesh, 2015), microswitches (Baghani, 2012), mass flow sensors (Nayfeh & Younis, 2005), and microfluidic devices (Dai, Wang, & Ni, 2015).

Since the thickness of microbeams is typically on the order of microns and sub-microns, the size-dependent static and dynamic behaviors have been experimentally verified (see, e.g., Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003; Liu et al., 2012). These size-dependent behaviors challenge the applicability of the theories of classical continuum mechanics. Since the classical continuum mechanics is not capable of predicting the size-dependent behavior of micro-structures (e.g., microbeams, microplates and microtubes), various new non-classical

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continuum theories, such as the strain gradient (Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012) and modified couple stress theories (Yang, Chong, Lam, & Tong, 2002) have been developed.

The available theoretical models regarding the size-dependent dynamical behavior of microbeams may be grouped into two: linear theoretical models (Ke, Wang, Yang, & Kitipornchai, 2012; Mohammad-Abadi & Daneshmehr, 2014; Rahaeifard & Ahmadian, 2015; Simsek & Reddy, 2013; Wang, Xu, & Ni, 2013) and nonlinear theoretical models (Xia, Wang, & Yin, 2010; Asghari, Kahrobaiyan, & Ahmadian, 2010; Farokhi & Ghayesh, 2015; Ghayesh, Amabili, & Farokhi, 2013; Ke et al., 2012). In the linear models, the size-dependent vibration characteristics of microbeams are predicted based on linear theories. Thus, the linear theoretical models are only valid for small deflections and could not predict the dynamic response of microbeams undergoing relatively large motions. The nonlinear theoretical models, however, has accounted for the system's possible nonlinearities (e.g., geometric nonlinearities due to the microbeam's deformation). Therefore, if the effect of nonlinearities on the deflection of microbeams is non-negligible, the nonlinear theoretical models are more reliable regarding the dynamical behavior of the system.

The literature on the static deformation and dynamic characteristics of microbeams using linear theoretical models is quite large. The microbeams were modeled either using Euler–Bernoulli beam (Park & Gao, 2006) or Timoshenko beam (Ma, Gao, & Reddy, 2008) assumptions. Both strain gradient (Kahrobaiyan et al., 2012) and modified couple stress (Yang et al., 2002) theories have been utilized for developing size-dependent beam models. Several researchers have also studied the linear vibrations of microbeams containing internal fluid flow (Dai et al., 2015; Rinaldi, Prabhakar, Vengallator, & Paidoussis, 2010; Wang, 2010; Yang, Ji, Yang, & Fang, 2014; Yin, Qian, & Wang, 2011).

The literature concerning the nonlinear dynamics of size-dependent microbeams is not large. Perhaps the earliest work contributed to this field is due to Asghari et al. (2010) and Xia et al. (2010). Asghari et al. (2010) developed a nonlinear size-dependent Timoshenko beam model based on the modified couple stress theory. The nonlinear behavior of the theoretical model is due to the presence of induced mid-plane stretching in the microbeam with two immovable supports. Xia et al. (2010) studied the static bending, postbuckling and free vibration of Euler–Bernoulli microbeams with pinned–pinned boundary conditions by constructing a nonlinear theoretical model using the modified couple stress theory. The results reported in Asghari et al. (2010) and Xia et al. (2010) showed that the nonlinear static deflection of the microbeam is smaller while the nonlinear frequency is higher than their counterparts predicted by linear theoretical models.

In the past 5 years, indeed, a series of papers have dealt with the nonlinear problems of microbeams, attempting to analyze the free vibrations (Ke et al., 2012) and forced vibrations (Ghayesh, Farokhi, & Amabili, 2013). For a cantilevered microbeam, the centerline may be assumed to be inextensible. It is therefore that only one single governing equation for lateral motions is required. For a microbeam fixed at both ends, as the inextensibility condition of centerline can no longer be applied, two governing equations are necessary: one is in the longitudinal and the other in the lateral direction. Since the nonlinear dynamics of cantilevered and supported microbeams has essential difference, they should be treated separately. To the authors' knowledge, the literature on the nonlinear dynamics of microbeams was mainly concerned with microbeams fixed at both ends. The literature on the nonlinear dynamics of cantilevered microbeams is very limited. There is a lack of nonlinear equations of motion for cantilevered microbeams with consideration of small length scale effect. This motivates the current work.

The objective of this study is to develop a microstructure-dependent nonlinear model and apply it to investigate the nonlinear dynamics of cantilevered microbeams with consideration of both size effect and large-deflection-induced nonlinearities. Firstly, a new, full nonlinear equation of motion is derived using the Hamilton's principle. Then, by virtue of Galerkin's method and modal truncation, the microbeam system is recast into a set of ordinary differential equations (ODEs). Based on these ODEs, the primary resonances of the first mode of the cantilevered microbeam are analyzed, showing size-dependent properties and the occurrence of softening-type nonlinear behavior. The quantitative dynamic responses of the microbeam are then studied in the case where a linear or nonlinear spring support is present. Numerical results show that the intermediate spring support could significantly affect the nonlinear responses of the microbeam system.

2. Full nonlinear equation of motion

The system under consideration is a slender microbeam having length L, cross-sectional area A, mass per unit length m and classical flexural rigidity EI. A schematic diagram for the physical system is shown in Fig. 1(a). The purpose of this section is to derive a complete governing differential equation for a cantilevered microbeam moving in a plane, when the motion is large.

As illustrated in Fig. 1(b), in the case of flexural vibrations of the microbeam, two coordinate systems may be introduced: the Eulerian (x,z) and the Lagrangian (x_0,z_0). Consider the undisturbed axis of the microbeam to be vertical, along the x_0-axis. Thus, the microbeam is initially lying along the x_0-axis and would vibrate in the (x_0,z_0) plane. It is assumed that the effect of gravity is non-negligible and the x_0-axis is in the direction of gravity. For a cantilevered microbeam, the centerline may be considered to be inextensible and the inextensibility condition may be expressed as (Paidoussis, 1998)

$$\left(\frac{\partial x}{\partial x_0}\right)^2+\left(\frac{\partial z}{\partial x_0}\right)^2=1 \tag{1}$$