International Journal of Engineering Science 94 (2015) 164–180

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Micromechanical models for time-dependent multiphysics responses of polymer matrix smart composites

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ARTICLE INFO

Article history:
Received 10 March 2015
Received in revised form 27 May 2015
Accepted 28 May 2015

Keywords:
Time-dependent multiphysics responses
Smart composites
Polymer matrix
Micromechanics

ABSTRACT

Stemming from the viscoelastic behavior of the polymer matrix, the effective material properties of smart composites with polymer matrix become time-dependent. This study has two focuses. Firstly, a micromechanics model for predicting the effective time-dependent properties of polymer matrix smart composites was developed. One important contribution of this model is that Laplace transformation and inversion commonly used for linear viscoelastic composites are avoided since all calculations are accomplished in the time domain. Secondly, by taking advantage of the predicted effective time-dependent properties a computation approach was proposed to simulate the effective multiphysics responses of multiphase smart composites subjected to multiphysics loadings. Excellent agreements are achieved between the predictions provided by the present models and ABAQUS.

1. Introduction

The smart composite consisting of piezoelectric and piezomagnetic constituents displays a magneto-electric coupling effect that is absent in constituents (Avellaneda & Harshe, 1994; Benveniste, 1995; Harshe, Dougherty, & Newnham, 1993; Huang, 1998; Huang, Liu, & Dai, 2000; Li, 2000; Nan, 1994; Van Run, Terrell, & Scholing, 1974; Wu & Huang, 2000). The magneto-electric coupling effects created by the interaction of piezoelectric phases and piezomagnetic phases has recently been extensively investigated due to their broad engineering applications (Aboudi, 2001; Huang & Kuo, 1997; Li & Dunn, 1998). Since the piezoelectric and piezomagnetic ceramics are brittle and susceptible to fracture, adding a polymer matrix into the two-phase electromagnetoelastic composite will increase the ductility and formability of the composites (Lee, Boyd, & Lagoudas, 2003). In order to predict the effective properties of this three-phase electromagnetoelastic composites, Lee, Boyd, and Lagoudas (2005) developed a finite element analysis-based micromechanics approach through averaging of the representative volume element (RVE) to determine the effective dielectric, magnetic, mechanical, and coupled-field properties of this composites as functions of the phase volume fractions, the fiber arrangements in RVE, and the fiber material properties with special emphasis on the poling directions of the piezoelectric and piezomagnetic fibers. In these models, the polymer matrix is treated as elastic solid.

It is well known that the polymer matrix exhibits strong time-dependent behavior, namely, viscoelastic behavior. Very recently, Azrar, Bakkali, and Aljinaidi (2014) developed a micromechanics model to predict the time and frequency dependent effective properties of viscoelectroelastic composites. Their modeling is based on the correspondence principle of linear viscoelectroelasticity combined with Mori–Tanaka micromechanical model. Many other researches (Aldraihem, Baz, &

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http://dx.doi.org/10.1016/j.ijengsci.2015.05.010
0020-7225/

Al-Saud, 2007; Batra, 2001; Li & Dunn, 2001; Muliana & Li, 2010) have also been conducted to investigate the viscoelectroelastic behavior of composites consisting of piezoelectric phase and polymer matrix. However, none of them is involved in the electromagnetoviscoelastic behavior of composites.

The *V*ariational *A*symptotical *M*ethod for *U*nit *C*ell *H*omogenization (VAMUCH) (Tang & Yu, 2008; Yu & Tang, 2007) is a finite element-based, general-purpose micromechanics code to perform homogenization of heterogeneous materials based on the variational asymptotic method (Berdichevsky, 1977). It can be used to calculate the effective fully-coupled, multiphysical material properties, including thermal, elastic, electric, and magnetic for arbitrary heterogeneous materials with arbitrary microstructure providing a unit cell (UC), or a representative volume element (RVE), can be identified. The heterogeneous material could compose of arbitrary number of constituents having full anisotropy. VAMUCH calculates the complete set of material properties within one analysis. If needed, VAMUCH can also recover the multiphysical fields within the microstructure.

In this paper, a micromechanics model was firstly constructed on the basis of VAMUCH modeling theory to predict the effective time-dependent multiphysics properties of viscoelastic polymer matrix smart composites. Then a computation approach was proposed to calculate the multiphysics responses of polymer matrix smart composites subjected to multiphysics loadings by taking advantage of the predicted effective time-dependent properties. The paper is organized as follows. Section 2 presents the theoretical formulations of the multiphysics behavior of smart materials subjected to constant strain, electric field, and magnetic field. The micromechanics formulations of effective time-dependent properties of polymer matrix smart composites given in Section 3 is developed based on the VAMUCH model for electro-magneto-elastic composites. The simulation approach of multiphysics responses of polymer matrix smart composites is presented in Section 4. In Section 5, numerical examples are employed to demonstrate the accuracy and capability of the proposed models. The conclusions are drawn in Section 6.

2. Theoretical equations for time-dependent multiphysics responses of smart materials

According to the Boltzmann superposition principle, the constitutive equations for the electro-magneto-viscoelastic material can be expressed in the entire time domain in the following way

$$\sigma_{ij}(t)=\int_{-\infty}^{t}(C_{ijkl}(t-\tau)\dot{\varepsilon}_{kl}(\tau)-e_{ijk}(t-\tau)\dot{E}_k(\tau)-q_{ijk}(t-\tau)\dot{H}_k(\tau))d\tau \tag{1a}$$

$$D_i(t)=\int_{-\infty}^{t}(e_{ikl}(t-\tau)\dot{\varepsilon}_{kl}(\tau)+k_{ik}(t-\tau)\dot{E}_k(\tau)+a_{ik}(t-\tau)\dot{H}_k(\tau))d\tau \tag{1b}$$

$$B_i(t)=\int_{-\infty}^{t}(q_{ikl}(t-\tau)\dot{\varepsilon}_{kl}(\tau)+a_{ik}(t-\tau)\dot{E}_k(\tau)+\mu_{ik}(t-\tau)\dot{H}_k(\tau))d\tau \tag{1c}$$

where $C_{ijkl}(t)$ is the stress relaxation stiffness; $e_{ikl}(t)$ and $q_{ijk}(t)$ are the piezoelectric and piezomagnetic relaxation tensors, respectively; $k_{ik}(t)$, $a_{ik}(t)$, and $\mu_{ik}(t)$ are the dielectric, magnetoelectric, and magnetic permeability relaxation tensors, respectively; $\sigma_{ij}(t)$ and $\dot{\varepsilon}_{kl}(\tau)$ are the stress tensor and strain rate tensor, respectively; $D_i(t)$ and $B_i(t)$ are the electric displacement and magnetic induction vectors, respectively; $\dot{E}_k(\tau)$ and $\dot{H}_k(\tau)$ are electric field and magnetic field rate, respectively.

The stress relaxation tests are performed at constant strains. If the electrical field and magnetic field are also constant, which means

$$\varepsilon_{kl}(t)=\begin{cases}0 & t<0\\ \varepsilon_{kl}^{cst} & t\geqslant 0\end{cases}\quad E_k(t)=\begin{cases}0 & t<0\\ E_k^{cst} & t\geqslant 0\end{cases}\quad H_k(t)=\begin{cases}0 & t<0\\ H_k^{cst} & t\geqslant 0\end{cases} \tag{2}$$

where "cst" means constant values that do not vary with time but may change with position.

Eq. (2) implies: $\lim_{t\to-\infty}\varepsilon_{kl}(t)=0,\ \lim_{t\to-\infty}E_k(t)=0,\ \lim_{t\to-\infty}H_k(t)=0.$

Then applying the integral by parts to Eq. (1), we can obtain

$$\sigma_{ij}(t)=\left(C_{ijkl}(0)-\int_0^t\frac{\partial C_{ijkl}(t-\tau)}{\partial\tau}\mathrm{d}\tau\right)\varepsilon_{kl}^{cst}-\left(e_{ijk}(0)-\int_0^t\frac{\partial e_{ijk}(t-\tau)}{\partial\tau}\mathrm{d}\tau\right)E_k^{cst}$$
$$-\left(q_{ijk}(0)-\int_0^t\frac{\partial q_{ijk}(t-\tau)}{\partial\tau}\mathrm{d}\tau\right)H_k^{cst}=C_{ijkl}(t)\varepsilon_{kl}^{cst}-e_{ijk}(t)E_k^{cst}-q_{ijk}(t)H_k^{cst} \tag{3a}$$

$$-D_i(t)=-\left(e_{ikl}(0)-\int_0^t\frac{\partial e_{ikl}(t-\tau)}{\partial\tau}\mathrm{d}\tau\right)\varepsilon_{kl}^{cst}-\left(k_{ik}(0)-\int_0^t\frac{\partial k_{ik}(t-\tau)}{\partial\tau}\mathrm{d}\tau\right)E_k^{cst}$$
$$-\left(a_{ik}(0)-\int_0^t\frac{\partial a_{ik}(t-\tau)}{\partial\tau}\mathrm{d}\tau\right)H_k^{cst}=-e_{ikl}(t)\varepsilon_{kl}^{cst}-k_{ik}(t)E_k^{cst}-a_{ik}(t)H_k^{cst} \tag{3b}$$

$$-B_i(t)=-\left(q_{ikl}(0)-\int_0^t\frac{\partial q_{ikl}(t-\tau)}{\partial\tau}\mathrm{d}\tau\right)\varepsilon_{kl}^{cst}-\left(a_{ik}(0)-\int_0^t\frac{\partial a_{ik}(t-\tau)}{\partial\tau}\mathrm{d}\tau\right)E_k^{cst}$$
$$-\left(\mu_{ik}(0)-\int_0^t\frac{\partial \mu_{ik}(t-\tau)}{\partial\tau}\mathrm{d}\tau\right)H_k^{cst}=-q_{ikl}(t)\varepsilon_{kl}^{cst}-a_{ik}(t)E_k^{cst}-\mu_{ik}(t)H_k^{cst} \tag{3c}$$