International Journal of Engineering Science 49 (2011) 397–410

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Scaling of turbulent separating flows

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ARTICLE INFO

Article history:
Received 21 May 2010
Accepted 13 December 2010

Keywords:
Law of the wall
Adverse pressure gradient
Separation
Turbulent boundary layer
Asymptotic theory

ABSTRACT

The present work investigates the scaling of the turbulent boundary layer in regions of adverse pressure gradient flow. For the first time, direct numerical simulation and experimental data are applied to the theory presented in Cruz and Silva Freire [Cruz, D. O. A., & Silva Freire, A. P. (1998). On single limits and the asymptotic behaviour of separating turbulent boundary layers. *International Journal of Heat and Mass Transfer, 41*, 2097–2111] to explain how the classical two-layered asymptotic structure reduces to a new structure consistent with the local solutions of Goldstein and of Stratford at a point of zero wall shear stress. The work discusses in detail the behaviour of an adaptable characteristic velocity (u_R) that can be used in regions of attached as well as separated flows. In particular, u_R is compared to velocity scales based on the local wall shear stress and on the pressure gradient at the wall. This is also made here for the first time. A generalized law of the wall is compared with the numerical and experimental data, showing good agreement. This law is shown to reduce to the classical logarithmic solution and to the solution of Stratford under the relevant limiting conditions.

1. Introduction

The early attempts at constructing theories for attached turbulent boundary layer flows in the asymptotic limit of large Reynolds number take as a central postulate the notion that the flow structure can be subdivided into two layers: (i) a wall viscous layer, in which the turbulent and laminar stresses are of comparable magnitude and (ii) a defect layer, in which the velocity profile may be expressed in terms of a small perturbation to the external flow solution. Both original notions were advanced by Prandtl (1925) and von Kármán (1930) through dimensional analysis. They naturally lead to a universal solution that has been shown by Millikan (1939) to have a logarithmic character and be dependent on velocity and length scales based on the friction velocity.

However, the action of a large adverse pressure gradient (APG) completely changes this picture, setting in a square-root velocity profile across the fully turbulent region that makes the previous scaling and asymptotic structures not suitable anymore (Stratford, 1959). In particular, at a point of flow separation the wall shear stress is zero so that none of the canonical theories (Bush & Fendell, 1972; Mellor, 1972; Yajnik, 1970) is valid.

Extensions of the Yajnik–Mellor (Mellor, 1972; Yajnik, 1970) theory to turbulent separation have been presented in literature, notably by Melnik (1989), who proposes a formulation based on a two parameter expansion. One parameter of fundamental importance to his developments, however, depends on a particular type of turbulent closure. This parameter is further used to regulate the order of magnitude of the pressure gradient term. Sychev and Sychev (1987) use the same

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doi:10.1016/j.ijengsci.2010.12.001

theoretical framework of Yajnik (1970), Mellor (1972) and Bush and Fendell (1972) but consider an extra layer where the internal friction forces, the pressure gradient and inertia forces balance each other. In their approach, no turbulence closure is required.

Many other different theories have been proposed in literature to explain the flow behaviour near to a separation point. However, no study satisfactorily elucidates the question of the scaling of mean velocity profiles in the entire domain of APG flows. As mentioned above, the appropriate near wall velocity scales for both attached and separated flows have long been proposed by Prandtl (1925) and von Kármán (1930), and Stratford (1959), respectively. Regrettably, the conditions under which one must switch to the other and their impact on the local flow solution and asymptotic structure have not been adequately discussed in literature.

The present work uses a Kaplun-limit analysis (Cruz & Silva Freire, 1998) to correlate directly the asymptotic structure of a separating flow with velocity and length scales ($\epsilon = u_R/u_e, u_R$ = reference velocity, u_e = external flow velocity) based on a cubic algebraic equation that expresses the balance between pressure and internal friction forces in the inner regions of the flow and explains in physical terms the correct limiting behaviour of the local scales. For the first time, velocity and length scales based on the local wall shear stress, the local wall pressure gradient and on a combination of both are comprehensively compared with numerical and experimental data in regions of attached, separated and reversed flow.

The work explains in a systematic way how the characteristic lengths of the wall viscous region $\hat{\epsilon}(\text{ord}(\hat{\epsilon}) = \text{ord}(1/\epsilon R)$, R = Reynolds number = $u_e l/\nu$, $l = (\rho u_e^2/(\partial_x p)_w), w =$ wall conditions) and of the fully turbulent region $\tilde{\epsilon}(\text{ord}(\tilde{\epsilon}) = \text{ord}(\epsilon^2))$ define an asymptotic structure that is valid throughout the flow region. Of special interest is an explanation about the relative change in order of magnitude of the characteristic lengths at a separation point. The work also shows that in the reverse flow region a y^2-solution prevails over most of the near wall region. This finding is opposed to the results reported in Simpson (1983), who proposes a log-solution.

The validity domain of a local solution previously developed to describe the velocity profile in the fully turbulent region of the flow (Loureiro, Soares, Fontoura Rodrigues, Pinho, & Silva Freire, 2007; Loureiro, Pinho, & Silva Freire, 2007; Loureiro, Monteiro, Pinho, & Silva Freire, 2008) is compared with the characteristic flow regions defined by the presently identified asymptotic structure. This is also made here for the first time. The local solution satisfies the limiting behaviour for attached (log-solution) as well as separated ($y^{1/2}$-solution) flows.

The current results are validated against three data sets: the flat-plate flows of Na and Moin (1998) and of Skote and Hennigson (2002) and the flow over a steep hill of Loureiro et al. (2007). These data were chosen for the very distinct geometry of flows they cover in addition to the possibility of analyzing the separation point region. The data of Na and Moin (1998) and of Skote and Hennigson (2002) were obtained through a direct numerical simulation of the Navier–Stokes equations. Therefore, they are very detailed but for a low Reynolds number range. The data of Loureiro et al. (2007) were obtained through LDA measurements; they cover a higher Reynolds number range and furnish wall shear stress, mean velocity and turbulence profiles.

2. Relevant scales for separating flows

The above remarks concerning the changes in scaling laws will now be given a brief analytical explanation.

2.1. Characteristic scales for attached flows

The two-layered model established by Prandtl (1925), von Kármán (1930, 1939) for attached flows considers that across the wall layer the total shear stress deviates just slightly from the wall shear stress. Hence, in the viscous layer a linear solution $u^+ = y^+$ follows immediately with $u^+ = u/u_*$, $y^+ = y/(\nu/u_*)$ and $u_* = \sqrt{\tau_w/\rho}$.

For the turbulence dominated flow region we may write:

$$\partial_y \tau_t = \partial_y(-\rho\overline{u'v'}) = 0. \quad (1)$$

A simple integration of the above equation implies that ord(u') = ord(v') = ord(u_*), where we have clearly considered the velocity fluctuations to be of the same order.

The analysis may proceed by taking as a closure assumption the mixing-length theory. A further equation integration yields the classical law of the wall for a smooth surface:

$$u^+ = \varkappa^{-1} \ln y^+ + A, \quad (2)$$

where $u^+ = u/u_*$, $y^+ = y/(\nu/u_*)$, $\varkappa = 0.4$, $A = 5.0$.

2.2. Characteristic scales for separated flows

The essential description of the physics of flow at a separation point has been given by Goldstein (1930, 1948) and by Stratford (1959). The action of an arbitrary pressure rise in the inner layer distorts the velocity profile implying that the gradient of shear stress must now be balanced by the pressure gradient.