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Lagrangian Coherent Structures in Tandem Flapping Wing Hovering

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Abstract

Lagrangian Coherent Structures (LCS) of tandem wings hovering in an inclined stroke plane is studied using Immersed-Boundary Method (IBM) by solving two dimensional (2D) incompressible Navier-Stokes equations. Coherent structures responsible for the force variation are visualized by calculating Finite Time Lyapunov Exponents (FTLE), and vorticity contours. LCS is effective in determining the vortex boundaries, flow separation, and the wing-vortex interactions accurately. The effects of inter-wing distance and phase difference on the force generation are studied. Results show that in-phase stroking generates maximum vertical force and counter-stroking generates the least vertical force. In-phase stroking generates a wake with swirl, and counter stroking generates a wake with predominant vertical velocity. Counter stroking aids the stability of the body in hovering. As the hindwing operates in the wake of the forewing, due to the reduction in the effective Angle of Attack (AoA), the hindwing generates lesser force than that of a single flapping wing.

Keywords: dragonfly, tandem wings, LCS, IBM

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1 Introduction

Vortex dynamics and flow separation at high Angle of Attack (AoA) play a significant role in the aerodynamics of insects, birds, and Micro Aerial Vehicles (MAVs). Force generation in flapping flight is coupled to the existence of Leading Edge Vortex (LEV)[1], momentum transfer, and entrainment of surrounding fluid by counter-rotating vortices. As a result, the identification and the analysis of vortex structures become significant. Shyy and Liu[2] gave a comprehensive review of the aerodynamics of flapping wings. Most of the vortex feature identification techniques like Q and lambda criteria[3] require a user-defined, preselected threshold to define the boundaries of vortices[4]. By applying concepts of dynamical systems theory to fluid motions, Haller and Yuan[5] proposed the Finite-Time Lyapunov Exponent (FTLE) fields to visualize the Lagrangian Coherent Structures (LCS). Haller and Peacock[6] reviewed LCS and its application to geophysical flows. Based on the work of Haller[5,7], Shadden *et al*.[8] presented an improvised definition of LCS as a ridge of FTLE field. Deformation tensor of the fluid is calculated over a finite time interval, and the maximum eigenvalue of the tensor represents the ridges of FTLE. LCS are the ridges of FTLE subjected to an additional hyperbolicity condition[9] to nullify the flux across the boundaries of the structures. FTLE obtained by integrating forward in time quantifies the separation between two nearby particles over the time interval. FTLE ridges, therefore, are curves along which particles are most prone to deviate from one another[10]. The ridges are called repelling LCS analogous to stable manifolds in dynamical systems theory. In contrast, if the integration is performed backward in time, then the ridges attract two particles which are separated by a distance in the beginning. The ridges are analogous to unstable manifolds and are called attracting LCS.

In the past decades, LCS have been utilized to visualize vortex structures in diverse fields. Peng and Dabiri[11] investigated the wake dynamics of both swimming and flying animals by examining both repelling and attracting LCS. Brunton and Rowley[12] used LCS to visualize vortical structures in the wake of a flat plate undergoing pitching and plunging. Wan *et al*.[13] studied the wake vortices in a plate undergoing harmonic and non-harmonic pitching and plunging using backward FTLE ridges. Eldredge and Chong[10] studied fluid

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transport and coherent structures of translating and flapping wings, and compared the change in the vortical structures for both rigid and exible wings. Yang *et al*.[14] visualized the repelling and attracting LCS of a starting vortex ring generated by a thin circular disk. Their results revealed a flux window between the attracting and repelling structures which entrains the shear flow into a vortex. Recently, Rosti *et al*.[15] performed Direct Numerical Simulation (DNS) of the flow around a pitching airfoil at high Reynolds number and visualized Kelvin-Helmholtz instabilities by employing backward FTLE ridges.

Dragon flies are one of the highly manoeuvrable flyers with independently moving fore- and hind wings. Lan and Sun[16] solved 2D incompressible Navier-Stokes equations on moving overset grids to study hovering elliptical foils at 0˚, 90˚, and 180˚ phase differences. Wang and Russell[17] proposed an idealized kinematics mimicking dragonfly kinematics and studied the power requirements in a dragonfly hovering. They found that counter-stroking utilizes minimal power and generates sufficient lift to keep an insect aloft. Specific power consumption in hovering reduces with elastic storage in the muscles (Shen and Sun[18]). Usherwood *et al*.[19] used robotic wing experiments to study the vorticity dynamics of a dragonfly flight and proved that a dragonfly employs wing phasing to remove swirl from the wake and improves the aerodynamic efficiency. Xiang *et al*.[20] performed a parametric analysis of corrugated tandem wings, and their results show that lift-drag ratio for the wings is only marginally affected by the corrugations. Broering and Lian[21] studied tan tandem wings pitching and plunging in a vertical plane, and showed that both inter-wing distance and wing phasing can be used to control the force generation. Most of the past studies have focused on tandem wings pitching and plunging in a vertical plane. Recently, Broering and Lian[22] extended the work to 3D and showed that at low Reynolds number, 2D simulations reasonably predict the unsteady mechanisms of force generation.

However, for the LCS, the effects of inter-wing distance and wing phasing on force generation have not been studied for systems mimicking dragonfly kinematics. In this paper, we study a virtual tandem flapping wing model performing idealized dragonfly kinematics in a quiescent fluid. Velocity fields are obtained by solving incompressible 2D Navier-Stokes equation using immersed boundary method. Backward FTLE ridges are used in conjunction with vorticity contours to study the effect of phase difference and inter-wing distance on the aerodynamics.

2 Governing equations and the numerical method

2.1 Immersed boundary projection method

Immersed boundary projection method proposed by Taira and Colonius[23] is used in the present study. Incompressible 2D Navier-Stokes equations are solved on a cartesian grid called Eulerian grid, $\mathcal{D}$, and a set of discrete Lagrangian points, ξ_k, represent the surface of the body, $\mathcal{B}$.

The governing equations are:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla p + \nu\Delta\boldsymbol{u} + \int_S \boldsymbol{f}(\boldsymbol{\xi}(s,t))\delta(\boldsymbol{\xi}-\boldsymbol{x})\mathrm{d}s, \quad (1)$$

$$\nabla\cdot\boldsymbol{u} = 0, \quad (2)$$

$$\boldsymbol{u}(\boldsymbol{\xi}(s,t)) = \int_S \boldsymbol{u}(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{\xi})\mathrm{d}\boldsymbol{x} = \boldsymbol{u}_B(\boldsymbol{\xi}(s,t)), \quad (3)$$

where $\boldsymbol{x} \in \mathcal{D}$ and $\boldsymbol{\xi}(s, t) \in \mathcal{B}$. The boundary $\mathcal{B}$, is parameterized by s, and moves at the velocity, $\boldsymbol{u}_B(\boldsymbol{\xi}(s, t))$. Staggered grid finite difference formulation is used to discretize the above equations with pressure at the center of the cell and velocity fluxes on the cell faces. Explicit second order Adams-Bashforth scheme is used to discretize the convective terms and implicit Crank-Nicholson scheme is used to discretize the viscous terms. The discretization yields a formal accuracy of second order in space and first order in time.

2.2 FTLE calculation

If $\boldsymbol{x}(t)$ represents fluid particles initialized over the flow field at a time t, $\boldsymbol{u}(\boldsymbol{x},t)$ represents the time dependent velocity, and $\boldsymbol{x}(t+T)$ represents the position of the particles after a time T, the trajectory of a fluid particle is obtained by:

$$\phi_t^{t+T}(\boldsymbol{x}) = \boldsymbol{x}(t+T) = x(t) + \int_t^{t+T} u(x,t)\mathrm{d}t. \quad (4)$$

An in finitesimal separation, $\delta\boldsymbol{x}$, at the initial time t changes to $\delta\boldsymbol{x}(t+T)$ by the relation:

$$\delta\boldsymbol{x}(t+T) = \nabla\phi_t^{t+T}(\boldsymbol{x})\delta\boldsymbol{x}, \quad (5)$$

where $\nabla\phi_t^{t+T}(\boldsymbol{x})$ represents the deformation gradient