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$\mathcal{H}_\infty$ control of 8 degrees of freedom vehicle active suspension system

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Abstract The main objective of this paper is to develop improved robust control techniques for an active suspension system utilizing an improved mathematical model. For that purpose, Euler Lagrange equation is used to obtain a mathematical model for vehicle active suspension system. The dynamics of driver's seat are included to get a more appropriate model. Robust $\mathcal{H}_\infty$ controllers are designed for the system to minimize the effect of road disturbances on vehicle and passengers. The performance of active suspension system is determined by measuring the heave acceleration of driver's seat and rotational acceleration of vehicle around its center of gravity. Effectiveness of the proposed controllers is validated by simulation results.

1. Introduction

According to ISO 2631-1 standard, if human body is continuously exposed to vibrations between 0.5 and 80 Hz, the risk of injury to vertebrae in lumbar region is drastically increased and may cause malfunction of the nerves connected to these segments (Chamseddine et al., 2006). Each one of us daily uses vehicles for traveling and the above fact shows the importance of comfortable ride and the need to minimize the vibrations caused by the irregularities in roads.

The suspension system of vehicle plays a vital role in improving ride quality and ride comfort. It connects vehicle's body to the tires and is a mean to transmit all forces between vehicle's body and road. The desirable characteristics of suspension system are better road handling and ride comfort (Appleyard and Wellstead, 1995). Poor ride quality and ride comfort can harm passengers, vehicle's body and the cargo inside (Granlund, 2008). So the suspension system should be designed to take into account all these constraints.

Suspension systems can be passive, semi-active or active. Passive suspension systems consists of energy storing elements along with dampers with fixed characteristics. Their performance is limited and can only be changed by changing the characteristics of dampers and springs. There is no control over the amount of energy added or dissipated. A heavily damped system will provide good road handling but poor ride

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quality, similarly, a lightly damped system will provide good ride quality but poor road handling. Most of the vehicles in the world are using this suspension system. Compared to that, a semi-active suspension systems allows controlled damping with fixed spring characteristics (Pionke and Bocik, 2011). The idea of semi-active suspension system first came to light in 1970s. These systems are designed to dissipate energy in a controlled manner by changing the damping, however, there is no way to add energy to the system with this suspension technique. Semi-active suspension systems provide better performance compared to passive suspension systems. An active suspension systems (ASS) consist of springs, dampers and force actuators which can dissipate or add energy to the system in a controlled way. Active suspension systems have obvious advantages over the passive and semi-active suspension systems because their actuator can be controlled by controllers to provide ride comfort to the passengers. These systems provide better compromise between road handling and ride quality. Sensors continuously monitor the operating conditions and control units control the active actuators using the information of sensors (Izawa et al., 1997).

Vehicle suspension system models have been proposed regularly with time. But the research work for the analysis and practical implementation of vehicle suspension system started back in 80s. In 1987, it was shown that both ride quality and road handling can be improved by reducing the un-sprung mass (Hrovat, 1988). Design of vehicle suspension systems for ride comfort for frequencies below body structure resonances is discussed by Sharp and Crolla (1987). Due to added advantage of active suspension system, several research articles have also appeared in this domain. The linear quadratic regulator (LQR) control and proportional derivative integral (PID) control techniques are applied to active suspension system in Darus and Enzai (2010). $\mathcal{H}_\infty$ control theory has been utilized to design controller for vehicle active suspension system by Amirifar and Sadati (2006), Yamashita et al. (1994), and Chen and Guo (2005). Adaptive control techniques for active suspension system are discussed by Sun et al. (2013b), Sun et al. (2013b), and Alleyne and Hedrick (1995). Effect of delays in actuator signals are handled by Li et al. (2014) and Du and Zhang (2007). Sampled data control of vehicle ASS is presented by Gao et al. (2010), an $\mathcal{H}_\infty$ approach is adopted therein. In addition to the model based control techniques, artificial intelligence based techniques have also been studied for vehicle ASS (Cao et al., 2010).

The design of vehicle active suspension system has been studied based on three widely used mathematical models, that is, the quarter-car model, half-car model and the full car model. In quarter car model, the suspension of single tire of the car is modeled. Most of the preliminary studies (Alleyne and Hedrick, 1995; Yamashita et al., 1994) and some recent articles (Li et al., 2014; Darus and Enzai, 2010) are based on the quarter car model. The half-car model is an improved mathematical model over the quarter car model. Here, bicycle model is used and vehicle is considered with two tires. Among others, Sun et al. (2013a) and Li et al. (2011) present a design of active suspension system based on half-car model of the system. Full car model of vehicle active suspension system has also been presented in literature and control techniques are presented therein, see for example Darus and Sam (2009), Sun et al. (2013b), and Yagiz and Hacioglu (2008) and references therein. In most of the aforementioned studies on active suspension system, the dynamics of the driver's seat have been ignored. However, these dynamics play an important part regarding ride comfort because passenger is directly affected by the behavior of the seat. Therefore, in some of the recent studies, the dynamics of driver's seat are also incorporated in the mathematical model and some control strategies are proposed (Aly and Salem, 2013; Rahmi, 2003; Guclu, 2004).

The main contribution of this work is that a more detailed derivation of mathematical model of full car active suspension system including dynamics of driver's seat are presented. Furthermore, $\mathcal{H}_\infty$ state feedback control and $\mathcal{H}_\infty$ dynamic output feedback control schemes are devised for the developed model to minimize the effect of terrain irregularities on passenger comfort. The effectiveness is demonstrated by simulation results.

The remainder of this paper is organized as follows. In Section 2, the mathematical model of eight degrees of freedom active suspension system of full vehicle model including driver's seat dynamics is developed using Euler Lagrange approach. Section 3 includes the design of robust $\mathcal{H}_\infty$ controllers for suspension system is discussed. These controllers increase passenger comfort by minimizing the effect of road disturbances. The idea of robust controllers is supported by designing state-feedback and dynamic output-feedback controllers. Simulations are carried out to extend the understanding of proposed controllers. Finally, some concluding remarks are presented in Section 4 for further research.

2. Modeling of system

The mathematical model of a full vehicle is developed using the famous Lagrangian equation. The active suspension system considered has eight degrees of freedom. The model of driver's seat is also taken into account because the damping of seats have a vital role in ride quality and comfort provided by the vehicle.

2.1. Euler Lagrange equation

To derive a mathematical model using Euler Lagrange equations, an energy function called *"Lagrangian energy function"* is defined for the system. The Lagrangian function for a system is the difference of the total kinetic energy and the total potential energy of the system. Based on this function, equations of motion for the system are derived by the following expression;

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i \tag{1}$$

where L is the Lagrangian energy function, D is the dissipation function of system, q_i is the generalized ith coordinate, $\dot{q}_i$ and Q_i is the force on i_{th} coordinate.

2.2. System description

The suspension system considered has eight degrees of freedom, that is, the vehicle's pitch angle, roll angle, displacement of driver's seat, displacement of vehicle's sprung mass and displacement of four unsprung masses. The numerical values for the parameters of the system are presented in Table 1. The derivation of mathematical model is simplified by considering the following assumptions.

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