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Dispersion study of SH-wave propagation in an irregular magneto-elastic anisotropic crustal layer over an irregular heterogeneous half-space

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Abstract This study investigates the effect of various parameters on the propagation of seismic SH-waves in a magneto-elastic anisotropic crustal layer with corrugated boundary surfaces, lying over a heterogeneous half-space. The shear elastic modulus and mass density of half-space are the exponential functions of depth. Inclusion of the concept of corrugated irregularity with magneto-elasticity in the anisotropic (Monoclinic) medium and heterogeneity in the half-space medium brings a novelty to the existing literature related to the study of SH-wave. The expression of general dispersion relation has been established in closed form by using suitable boundary conditions. The effects of magneto-elastic coupling, heterogeneity, corrugation, undulatory and position parameters on the phase velocity of SH-wave have been computed numerically and demonstrated graphically. Moreover, different cases of free and common surface corrugations are studied which serve as a focal theme of the study.

1. Introduction

Regulation of seismic waves like propagation, reflection, transmission and refraction through an elastic layered media (coupled with different fields) is totally controlled by the properties and divergent irregular contact surfaces of the layered media. The study of seismic waves and their characteristics is often carried out to identify the structure and dynamics of the Earth's interior as well as to detect the epicentre of earthquakes. The analysis of seismic wave propagation in some complex media is also helpful in the exploration of natural resources buried inside the Earth's surface, e.g., oils, gases, minerals, crystals, metals and other useful hydrocarbons. It infers lot of information about the velocity of the wave, inward peculiarities of the media and forms the core tool of geophysical and earthquake sciences.

The magnetic and elastic properties of magneto-elastic materials depend on each other. The Earth's crust is made of the great diversity of igneous, metamorphic and sedimentary rocks. These rocks are capable to generate magnetic field due to the presence of some ferromagnetic minerals like iron, nickel,

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cobalt, etc. Hence, these rocks can be considered as the magneto-elastic body. The investigation into the SH-wave propagation in magneto-elastic bodies is quite important for the development of fundamental work on the mechanics of a magnetic body. Moreover, it may be used in various areas of science, engineering technology such as seismology, defectoscopy, geophysics, astrophysics and geo-tectonics. Keeping in view the significance of seismic wave propagation in magneto-elastic bodies, the considerable amount of investigations is carried out by several researchers. Notable among them are Abd-Alla et al. (2016), Said (2016), Majhi et al. (2016), Calas et al. (2008), Othman and Song (2006), Chattopadhyay and Singh (2014), Song et al. (2006), Abo-Dahab et al. (2015), Singh et al. (2016a) and Mahmoud (2016). Matinfar et al. (2015) observed the interaction of electromagnetic wave with electron using the variational iteration method.

It is well-known fact that the interface between any two adjacent layers of the Earth is very complicated and irregular in nature. These irregular interfaces may be in the shape of corrugation (undulated), rectangular, parabolic or much complicated and work as a catalyst in affecting the propagation behaviour of SH-waves. Some notable references related to the study of wave phenomena in/at irregular boundary surfaces are Vishwakarma and Xu (2016a), Singh and Lakshman (2016), Kundu and his co-workers (Kundu et al., 2014, 2016a), Singh et al. (2016b), Tomar and Kaur (2007) and Chatterjee et al. (2015).

The subject of wave propagation in a heterogeneous elastic medium is of great interest since long because of continuous change in the elastic properties of the material. These studies were recorded in several treatises including Birch (1952) and Bullen (1940). Subsequently, Vishwakarma and Xu (2016b), Daros (2013), Zhou et al. (2014), Sahu et al. (2014) and Kundu et al. (2016b) discussed for seismic waves in various types of heterogeneous media.

The present study investigates the affected behaviour of horizontally polarized shear wave (i.e. SH-wave) propagation in an anisotropic crustal layer of finite thickness, lying over a heterogeneous half-space. The boundary surfaces of considered structure are corrugated irregular. The crustal anisotropic layer has been regarded as a perfect conductor. The shear elastic modulus and mass density of the half-space have been considered in terms of the exponential function of depth i.e. $\mu_2 = \mu_0 e^{az}$ and $\rho_2 = \rho_0 e^{bz}$, where a and b are heterogeneous parameters. The effects of magneto-elastic coupling, upper free surface corrugation, common surface corrugation, heterogeneities and some geometrical parameters (undulatory and position) on the phase velocity of SH-wave against wave number have been shown by several graphs. This investigation for the propagation of SH-waves may be of importance when such types of waves are propagated on the Earth's surface where corrugated irregularity together with the magneto-elasticity, anisotropy and heterogeneity are present.

2. Governing equations

The Cartesian co-ordinate system has been considered in such a way that x-axis is in the direction of wave propagation and z-axis is vertically downwards. A magneto-elastic anisotropic crustal layer, $M_1 : [\lambda_1(x) - h] \leqslant z \leqslant \lambda_2(x)$ with corrugated boundary surfaces is taken in such a way that it lying over a heterogeneous half-space, $M_2 : \lambda_2(x) \leqslant z \leqslant \infty$. Here, h is the thickness of magneto-elastic anisotropic crustal layer; $\lambda_1(x)$ and $\lambda_2(x)$ are continuous and periodic functions of x independent of y, representing the corrugated boundaries of free and common surfaces as shown in Fig. 1. The appropriate Fourier series expansion of these functions can be given as (Singh, 2011; Vishwakarma and Xu, 2016a)

$$\lambda_j(x) = \sum_{n=1}^{\infty}(\lambda_n^j e^{in\alpha x} + \lambda_{-n}^j e^{-in\alpha x}), \quad j = 1, 2. \tag{1}$$

here, λ_n^j and λ_{-n}^j are the coefficients of Fourier series expansion of order n, such that

$$\lambda_{\pm n}^j = \begin{cases} \frac{a_j}{2}, & \text{for} \quad \mathrm{n} = 1 \\ \frac{A_n^j \mp iB_n^j}{2}, & \text{for} \quad \mathrm{n} = 2, 3, 4\ldots \end{cases}; j = 1, 2,$$

where A_n^j and B_n^j are the cosine and sine coefficients of Fourier series expansion of order n. In view of above expressions of λ_n^j and λ_{-n}^j, Eq. (1) leads to Tomar and Kaur (2007)

$$\lambda_j(x) = a_j \cos(\alpha x) + \sum_{n=2}^{\infty}\left[A_n^j \cos(n\alpha x) + B_n^j \sin(n\alpha x)\right], \; j = 1, 2. \tag{2}$$

The corrugated upper and common boundary surfaces of the concerned problem can be expressed by only one cosine term $\lambda_1 = a_1 \cos(\alpha x)$ and $\lambda_2 = a_2 \cos(\alpha x)$ for the wavelength $\frac{2\pi}{\alpha}$ of corrugation, where a_1 and a_2 are amplitudes of the uppermost and common corrugated surfaces and α is wave number of corrugation.

2.1. SH-wave in magneto-elastic anisotropic layer

Let $\vec{u}$ is the displacement vector field for anisotropic magneto-elastic layer whose components u_i are u_1, v_1 and w_1 in x, y and z directions respectively. Therefore, equations of motion for an anisotropic magneto-elastic layer in the presence of electromagnetic force $(\vec{J} \times \vec{B})$ that is Lorentz force are

$$\tau_{ij,j} + (\vec{J} \times \vec{B})_i = \rho_1 \frac{\partial^2 u_i}{\partial t^2} \quad i, j = 1, 2, 3, \tag{3}$$

here $(\vec{J} \times \vec{B})_i$ are the components of force $(\vec{J} \times \vec{B})$ in the i_{th} direction, $\vec{J}$ is the electric current density, $\vec{B}$ is the magnetic induction vector and ρ_1 is mass density. The stress–strain relations for the anisotropic material in xz-plane are Altenbach et al. (2004)

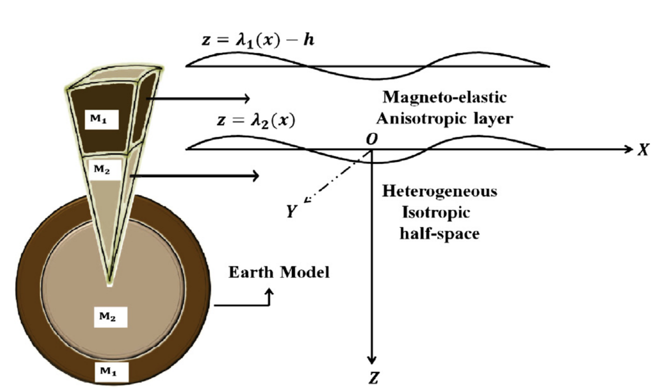

Fig. 1 Geometry of the problem,