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Some important classes of neighbor balanced designs in linear blocks of small sizes

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ABSTRACT

Neighbor balanced designs are useful to balance out the neighbor effects in field of agriculture, serology, agro forestry, industry, etc. In most of the agriculture experiments blocks are formed in a line and therefore, neighbor balanced designs are required in linear blocks. In this article some classes of first order neighbor balanced designs are presented in linear blocks of size three and four. A method to construct the second order neighbor balanced designs through two minimal first order neighbor balanced designs in linear binary blocks of size three is also developed here.

1. Introduction

If v treatments are arranged in b linear blocks of size k such that each unordered pair of adjacent treatments appears an equal number of times, say λ'_1, designs are called first order neighbor balanced designs in linear blocks. If $\lambda'_1 = 1$ then such designs are called minimal first order neighbor balanced designs in linear blocks. Kiefer and Wynn (1981) introduced an algorithm to construct the neighbor balanced designs (NBDs) in complete linear blocks. Cheng (1983) generated NBDs in linear blocks for different cases. Azais et al. (1993) constructed NBDs in complete blocks, in $k = v - 1$ and partially neighbor balanced designs in linear blocks. Jacroux (1998) constructed NBDs for all v having blocks of size 3 which are efficient under standard intrablock analysis as well as when experimental units adjacent within blocks are correlated. Tomar et al. (2005) constructed neighbor balanced block designs using Mutually Orthogonal Latin Squares (MOLS) and compared their designs with complete block designs balanced for neighbor effects. Ahmed (2010) constructed NBDs in linear blocks for k even, k odd &two different block sizes k_1 and k_2. Ahmed and Akhtar (2011) constructed NBDs in linear blocks of equal sizes for (i) $v = 4i + 1$, i integer, $k = 3$ with $\lambda' = 1$, (ii) $v = 2i + 1$, i (>1) odd, $k = 3$ with $\lambda' = 2$, and (iii) $v = 2i + 1$ (prime) and $k < v$. They also constructed these designs in linear blocks of unequal sizes for (i) $v = 4i–1$; in $k_1 = 3$ and $k_2 = 2$, $\lambda' = 1$, (ii) $v = 4i + 2$; in $k_1 = 3$ and $k_2 = 2$. Ahmed et al. (2013) developed some infinite series to generate minimal neighbor balanced designs for two and three different sizes in linear blocks. They also constructed generalized neighbor designs (GN_2-designs) in proper linear blocks. They developed following infinite series of minimal neighbor balanced designs in linear blocks. Shahid et al. (2017) constructed some important classes of generalized neighbor designs in linear blocks for four different cases. Minimal designs are always considered as the most economical. In this article, some infinite series are developed to generate the first order neighbor balanced designs for linear blocks of size 3 and 4. Catalogues are also presented of proposed designs. Two

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series are also developed to generate second order neighbor balanced designs through two minimal first order neighbor balanced designs in linear binary blocks of size three. These designs are constructed using method of cyclic shifts which is explained in Section 2. It is important to note that all proposed neighbor designs, in this article, are assumed balanced for non-directional neighbor effect. Non-directional neighbor effect means same effect from right or left neighbor and direction of neighbor does not matter.

2. Models and notations

Consider a class of designs $\Omega_{(v,b,k)}$ with v treatments grouped in b circular blocks of k experimental units per block.

Definition 1. A NBD at distance 1 is a binary design in which each treatment appears as first order neighbor λ' times to all other treatments.

Definition 2. A NBD at distance 2 is a binary design in which each treatment appears as first order neighbor λ'_1 times to all other treatments and each treatment appears as second order neighborλ'_2. times to all other treatments.

NBD at distance 1 & 2 are constructed under the following models 1 & 2 respectively:

$$y_{ij} = \mu + \beta_i + \tau_{d(i,j)} + \lambda_{1d(i,j\pm1)} + \varepsilon_{ij} \quad i = 1,2,\cdots,b; j = 1,2,\cdots,k \tag{1}$$

$$y_{ij} = \mu + \beta_i + \tau_{d(i,j)} + \lambda_{1d(i,j\pm1)} + \lambda_{2d(i,j\pm1)} + \varepsilon_{ij} \quad i = 1,2,\cdots,b;\ j = 1,2,\cdots,k \tag{2}$$

Models (1) and (2) can be written in vector as

$$\mathbf{Y_d} = \mathbf{1}\mu + \mathbf{B}\beta + \mathbf{T_d}\tau + \mathbf{U_{1d}}\lambda_1 + \varepsilon \tag{3}$$

$$\mathbf{Y_d} = \mathbf{1}\mu + \mathbf{B}\beta + \mathbf{T_d}\tau + \mathbf{U_{1d}}\lambda_1 + \mathbf{U_{2d}}\lambda_2 + \varepsilon \tag{4}$$

where $\mathbf{Y}$ and $\mathbf{1}$ are vectors of observations and 1's each of order $(bk \times 1)$ respectively, $\mathbf{T_d}$, $\mathbf{U_{1d}}$ and $\mathbf{U_{2d}}$ are incidence matrices of order $(bk \times v)$ for treatment, first order neighbor and second order neighbor effects respectively, and $\mathbf{B}$ is the $(bk \times b)$ incidence matrix for block effects. The vectors $\beta, \tau, \lambda_1, \lambda_2$ are the parameters of respective effects. ε is the vector of error term with mean vector $\mathbf{0}$ and variance-covariance matrix $\sigma^2\mathbf{I}$. The complete information matrix for model (3) is

$$\mathbf{C}_d = \begin{bmatrix} \mathbf{1}'\mathbf{1} & \mathbf{1}'\mathbf{B} & \mathbf{1}'\mathbf{T}_d & \mathbf{1}'\mathbf{U}_{1d} \\ \mathbf{B}'\mathbf{1} & \mathbf{B}'\mathbf{B} & \mathbf{B}'\mathbf{T}_d & \mathbf{B}'\mathbf{U}_{1d} \\ \mathbf{T}'_d\mathbf{1} & \mathbf{T}'_d\mathbf{B} & \mathbf{T}'_d\mathbf{T}_d & \mathbf{T}'_d\mathbf{U}_{1d} \\ \mathbf{U}'_{1d}\mathbf{1} & \mathbf{U}'_{1d}\mathbf{B} & \mathbf{U}'_{1d}\mathbf{T}_d & \mathbf{U}'_{1d}\mathbf{U}_{1d} \end{bmatrix}$$

where $\mathbf{1}'\mathbf{1} = bk = rv = n$, For an equireplicate designs with constant block size k; $\mathbf{B}'\mathbf{B} = kI_b$ and $\mathbf{T}'_d\mathbf{T}_d = rI_v$, $\mathbf{T}'_d\mathbf{B} = N$, $\mathbf{U}'_{1d}\mathbf{B} = 2N$. Let $\mathbf{T}'_d\mathbf{U}_{1d} = L$ and $\mathbf{U}'_{1d}\mathbf{U}_{1d} = M$, the complete information matrix becomes

$$\mathbf{C}_d = \begin{bmatrix} n & \mathbf{1}'\mathbf{B} & \mathbf{1}'\mathbf{T}_d & \mathbf{1}'\mathbf{U}_{1d} \\ \mathbf{B}'\mathbf{1} & kI_b & N' & 2N' \\ \mathbf{T}'_d\mathbf{1} & N & rI_v & L \\ \mathbf{U}'_{1d}\mathbf{1} & 2N & L & M \end{bmatrix}$$

After imposing restrictions and simplification, the joint information matrix for treatment and neighbor effects is

$$\mathbf{C}_{\tau u} = \begin{bmatrix} \mathbf{T}'_d\mathbf{T}_d - (\mathbf{T}'_d\mathbf{B}_d\mathbf{K})^{-1}(\mathbf{B}'_d\mathbf{T}_d) & \mathbf{T}'_d\mathbf{U}_{1d} - (\mathbf{T}'_d\mathbf{B}_d\mathbf{K})^{-1}(\mathbf{B}'_d\mathbf{U}_{1d}) \\ \mathbf{U}'_{1d}\mathbf{T}_d - (\mathbf{U}'_{1d}\mathbf{B}_d)\mathbf{K}^{-1}(\mathbf{B}'_d\mathbf{T}_d) & \mathbf{U}'_{1d}\mathbf{U}_{1d} - (\mathbf{U}'_{1d}\mathbf{B}_d)\mathbf{K}^{-1}(\mathbf{B}'_d\mathbf{U}_{1d}) \end{bmatrix}$$

$$\mathbf{C}_{\tau u} = \begin{bmatrix} rI_v - (1/k)NN' & L - (2/k)NN' \\ L - (2/k)NN' & M - (4/k)NN' \end{bmatrix}$$

Then information matrix for treatment is

$$\mathbf{C}_\tau = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{B}'$$

where

$$\mathbf{A} = rI_v - (1/k)NN'$$

$$\mathbf{B} = L - (2/k)NN'$$

$$\mathbf{D} = M - (4/k)NN'$$

Here NN' is the treatment concurrence matrix whose diagonal elements are repetitions of each treatment and off-diagonal elements are the number of times two treatments appear together in same blocks. L is the incidence matrix of treatments versus neighbors (left and right). Diagonal elements of L matrix, for a design in which no treatment appears as neighbor to itself, are zero and off-diagonal matrix are the number of times a pair of treatments appear as neighbor to each other in same blocks. For further detail see Iqbal et al. (2006, 2009). To achieve a NBD, all off-diagonal elements of matrix L must be same. If off-diagonal elements of matrix L contain two or more distinct values, the design is known as generalized neighbor design. Similarly, if off-diagonal elements of concurrence matrix NN' are same then the design is BIBD otherwise PBIBD.

Information matrix for neighbor effect in model (3) and information matrices for model (4) can be derived accordingly.

According to Hinkelmann and Kempthorne (2005), average variance of treatment contrast is a function of information matrix as

$$\underset{i\neq i'}{av}.\mathrm{var}(\hat{\tau}_i - \hat{\tau}_{i'})_{IBD} = 2\sigma^2_{e(IBD)}(v-1)^{-1}\sum_{i=1}^{v-1} d_i^{-1}$$

where d_i is the ith eigenvalue of $\mathbf{C}_\tau$. Eigen values of $\mathbf{C}_\tau$ can be calculated using R-language.

3. Method of construction and efficiency factor

3.1. Method of cyclic shifts

Method of cyclic shifts introduced by Iqbal (1991) is simplified here to construct neighbor balanced designs only in linear blocks". v treatments are labeled as 0, 1, 2, ..., $v-1$ under rule I and II below.

Rule I: Let $\underline{S}_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$ be a set of shifts where $1 \le q_{ji} \le v-1$. A design is first order neighbor balanced designs in linear blocks if each element of $\underline{S}_j$ along with its complement contains all elements 1, 2, ..., $v-1$ equally often, say, λ_1 times. In Rule I, complement of q_i is $v - q_i$.

Rule II: Let $\underline{S}_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-2)}]$t be a set of shifts where $1 \le q_{ji} \le v-2$. A design is first order neighbor balanced designs in linear blocks if each element of $\underline{S}_j$ along with its complement contains all elements 1, 2, ..., $v-2$ equally often, say, λ_1 times. In Rule II, complement of q_i is $v - 1 - q_i$.