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Original Article

Dynamic pressure change in a rotating, laterally oscillating cylindrical container

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Abstract

We examined wave phenomena pertinent to water in a rotating, laterally oscillating cylindrical container. In particular, we measured the time-dependent dynamic water pressure and pressure change by fast Fourier transform analysis. The swirling of water in the container had three frequency components; the frequency responses of each frequency component are reported herein. When swirling occurs in a rotating cylindrical container, it was found that the wave rotating in the same direction as the rotation of the cylindrical container and the wave rotating in the opposite direction to the cylindrical container exist at the same time. The swirling direction was determined by the relationship of these magnitude.

Keywords: Sloshing; Dynamic pressure; Cylindrical container; Rotation; Oscillation; Swirling.

1. Background

Sloshing, which is severe liquid agitation in a container, is a consequence of externally applied oscillation to the liquid. This is problematic in petroleum tanks and liquefied natural gas tankers; for example, the aftereffect of an earthquake or rolling tankers. Sloshing may lead to swirling [1], wherein a free surface rotates around the central axis of an axisymmetric container. Ibrahim [2] reported the theoretical, experimental, and numerical research pertinent to sloshing.

The stability and control of a rocket (or spacecraft) depends on swirling and other fluid dynamic behavior. Swirling is extremely dangerous in heavily fuel-laden rockets and missiles because it diverts trajectory. Yam et al. [3] investigated the stability of a spinning axisymmetric rocket exhibiting dissipative internal fluid motion. Bauer and Eidel [4] and Zhang et al. [5] examined free-surface oscillations in a slowly spinning cylindrical container partially filled with a viscous fluid. Ohaba et al. [6] investigated the frequency response of a liquid surface in a rotating, laterally oscillating cylindrical container using a capacitance wave-height meter. These studies indicate that cylindrical container rotation stabilizes swirling; nevertheless, detailed experiments have not yet been conducted.

In this study, we measure the time-dependent dynamic pressure of water in a rotating, laterally oscillating cylindrical container. We verified that the time-dependent dynamic pressure is proportional to free surface displacement from our previous measurements [7]. Moreover, we investigated the frequency components of swirling through fast Fourier transform (FFT) analysis of the time-dependent dynamic pressure change.

2. Experimental configuration

Figs. 1 and 2 show a schematic of the experimental apparatus and a cylindrical container, respectively. The cylindrical container is composed of Plexiglas (99 and 200 mm inner diameter and height, respectively). The rotating cylindrical container is connected to a motor fixed to the oscillating table. The table sinusoidally oscillates in the horizontal direction. Therefore, the cylindrical container laterally oscillates while rotating. We embedded the pressure sensor (12 mm above the

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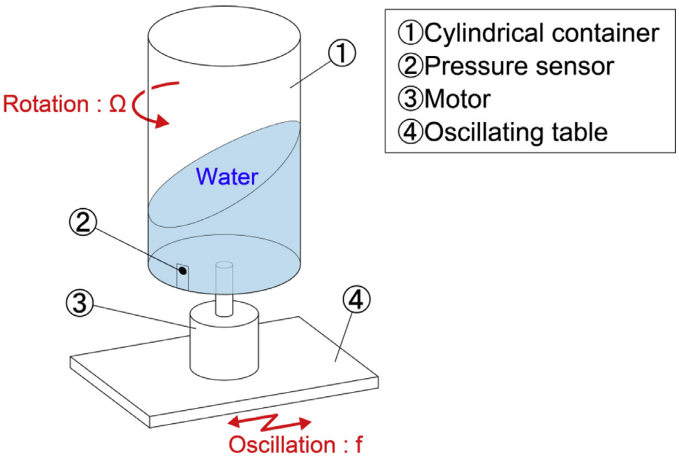

Fig. 1. Experiment apparatus.

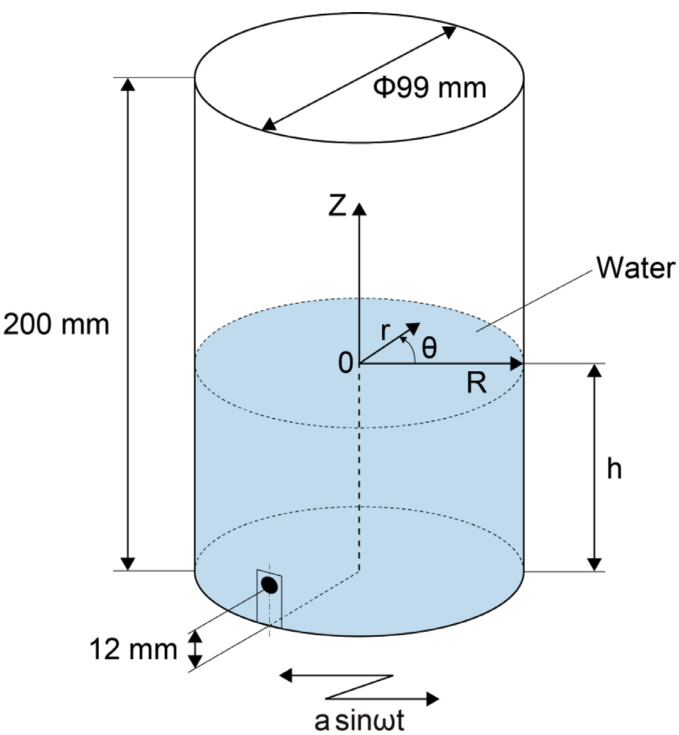

Fig. 2. Cylindrical container.

bottom wall) in the inner wall of the cylindrical container (Fig. 2). We measured the fluid dynamic pressure over the course of 50 oscillations of forcing frequency f. We increased f in increments of 0.01 Hz and measured the fluid dynamic pressure as previously described. The fluid dynamic pressure fluctuates when sloshing occurs. In one period of lateral oscillation of the cylindrical container, the difference between the maximum and minimum pressure is ΔP, measured 50 times for each f. We varied the forcing frequency f from 1.0 to 5.0 Hz. The amplitude of the lateral oscillation $a = 1.0$ mm, the rotating frequency of the cylindrical container $\Omega = 1.0$ Hz, and the water depth $h = 50$ mm.

3. Theoretical approach

Researchers have only begun to theoretically investigate sloshing-pertinent phenomena in a rotating cylindrical container. As a first step toward understanding these phenomena, we analyzed sloshing in the absence of rotation.

Fig. 2 shows our analytical model. Herein, we report z relative to the static level of the free surface. $\theta = 0$ indicates the direction of the forced oscillation. R is the radius of the cylindrical container, and ω is the angular frequency of the oscillating table. Assuming irrotational flow and an incompressible fluid, the unsteady irrotational Bernoulli equation for $z = \eta(r, \theta, t)$ is given by

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}|\nabla \phi|^2 + \frac{p}{\rho} + gz = a\omega^2 r \cos\theta \sin\omega t \tag{1}$$

where ϕ, p, ρ, and g are velocity potential, water pressure, water density, and gravitational acceleration, respectively. For infinitesimally small waves, we assumed negligible $|\nabla\phi|^2$. Using Eq. (1), the kinematic and dynamic free surface conditions are given by the following equations:

$$\frac{\partial \eta}{\partial t} = \left(\frac{\partial \phi}{\partial z}\right)_{z=0} \tag{2}$$

$$\left(\frac{\partial^2 \phi}{\partial t^2} + g\frac{\partial \phi}{\partial z}\right)_{z=0} = a\omega^3 r \cos\theta \cos\omega t \tag{3}$$

Boundary conditions on the bottom and side walls are given by

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=-h} = 0 \tag{4}$$

$$\left(\frac{\partial \phi}{\partial r}\right)_{r=R} = 0 \tag{5}$$

We solved the continuity equation $\nabla^2\phi = 0$ using boundary conditions (2)–(5). Eq. (6) describes the velocity potential

$$\phi = \sum_{M=1}^{\infty}\sum_{N=1}^{\infty}\Big\{A_m \cosh\left[k_{mn}(z+h)\right]J_m(k_m r)\cos(m\theta + \delta_{mn})\cos(\omega_{mn}t + {}_{mn}) + \frac{\omega_{mn}^2\omega^3}{\omega_{mn}^2 - \omega^2} \times \frac{ra\cos\theta\cosh\left[k_{mn}(z+h)\right]}{gk_{mn}\sinh(k_{mn}h)}\cos\omega t\Big\} \tag{6}$$

where m and n are natural numbers ($m = 1,\ 2,\ 3\cdots;\ n = 1,\ 2,\ 3\cdots$) that represent vibrational modes. J_m is the Bessel function of the first kind of order m. A_{mn}, δ_{mn}, and ϵ_{mn} are arbitrary constants. Herein, k_{mn} is a constant that satisfies the following equation:

$$\left[\frac{d}{dr}J_m(k_{mn}r)\right]_{r=R} = 0 \tag{7}$$

In addition, ω_{mn} is the characteristic angular frequency given by

$$\omega_{mn} = \sqrt{gk_{mn}\tanh(k_{mn}h)} \tag{8}$$

Eqs. (7) and (8) yield the resonant frequency, 2.97 Hz; $R = 49.5$ mm and $h = 50$ mm.