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Original Article

An analytical method for space–time fractional nonlinear differential equations arising in plasma physics

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Abstract

Here, a new fractional sub-equation method with a fractional complex transform is proposed for constructing exact solutions of fractional partial differential equations arising in plasma physics in the sense of modified Riemann–Liouville derivative, which is the fractional version of the known $\frac{D_\xi^\alpha G(\xi)}{G(\xi)}$ method. To illustrate the validity of this method, we apply it to the space–time fractional KdV equation on the dust ion acoustic waves in dusty plasma and space–time Boussinesq fractional equation. The proposed approach is efficient and powerful for solving wide classes of nonlinear evolution fractional order equations. The solutions obtained here are new and have not been reported in former literature.

Keywords: New frcational subequation method; Fractional complex transformation; Riemann–Liouville derivative; Exact solutions.

1. Introduction

Dusty plasma is ubiquitous in most space and astrophysical space environments, namely, lower and upper mesosphere, cometary's tails, planetary rings, planetary magnetosphere, interplanetary spaces, interstellar media, etc. [1–3]. Dusty plasma applications ranged from astrophysics to strongly coupled dusty plasmas and dusty plasma crystals [4]. Akhtar et al. [5] investigated Sagdeev's pseudo potential in un-magnetized two types of dust fluids (one cold and the other hot) in the presence of Boltzmann ions and electrons.

The dusty plasma medium with dissipative properties supports the existence of shock waves instead of solitons. The dust fluid dissipation can be caused by dust–dust collision, Landau damping, dust charge fluctuation and dust fluid viscosity, which would modify the wave profile structures [6–9]. Elwakil et al. [10] studied the effect of non-thermality of ions on the nature of dust-acoustic waves solitons and energy in two types dust fluid in un-magnetized, collisionless dusty plasma consisting of electrons, charged non-thermal ions, hot and cold dust grains. Nakamura et al. [11] observed shock solitary waves in unmagnetized dusty plasma. They showed that the development of the wave shock is due to KdV-Burgers equation. Mamun et al. [12] have studied the dust-ion-acoustic shock and solitary waves in dusty electronegative plasma and discussed the basic properties of the shock and solitary waves, which are associated with the presence of positive ion dynamics and dust charge fluctuation.

The fractional calculus was planted over 300 years ago. Science than, the subject of factional calculus [13–19] is a rapidly growing field of research, at the interface between chaos, probability, differential equations, and mathematical physics. In recent decades, fractional differential equations have gained much attention as they are widely used to describe various complex phenomena in many fields such as the fluid flow, signal processing, control theory, systems identification, biology and other areas. These equations appear in a wide great array discretization span, Jumarie's defined the fractional derivative in a variety of of contexts, such as physics, biology, engineering, signal processing, systems identification, control theory, finance and fractional dynamics [20–26]. Many powerful and efficient methods have been proposed to obtain numerical and exact solutions of the fractional PDEs [24] etc.

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In this paper, we will employ the new fractional sub-equation method to establish exact solutions for fractional partial differential equations (FPDEs), which is based on the following fractional ordinary differential equation as [27,28]

$$AG(\xi)D_\xi^{2\alpha}G(\xi)-BG(\xi)D_\xi^{\alpha}G(\xi)-C[D_\xi^{\alpha}G(\xi)]^2-EG(\xi)^2=0, \quad (1)$$

where $(D_\xi^{\alpha}G(\xi))$ denotes the modified Riemann–Liouville derivative of order α for $G(\xi)$ with respect to ξ, and A, B, C and E are constants to be determined later.

The rest of this paper is organized as follows. we present some definitions and properties of JumarieÆs modified Riemann–Liouville derivative and the expression for $\left(\frac{D_\xi^\alpha G(\xi)}{G(\xi)}\right)$ related to Eq. (1). In Section 2, we give the description of the fractional sub-equation method for solving FPDEs. Then in Section 2, we apply this method to establish exact solutions for the space–time fractional KdV on the dust ion acoustic waves in dusty plasma and space–time Boussinesq fractional equation. Some conclusions are presented at the end of the paper.

Jumarie's Riemann–Liouville derivative of order α is defined as

$$D_t^\alpha f(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t(t-\xi)^{-\alpha}(f(\xi)-f(0))d\xi, 0<\alpha<1 \quad (2)$$

$$D_t^\alpha f(t)=(f^n(t))^{\alpha-n}, n<\alpha<n+1, n>1 \quad (3)$$

The properties of the fractional Riemann–Liouville are summarized as

$$D_t^\alpha t^\gamma=\frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)}t^{\gamma-\alpha}, \gamma>0, \quad (4)$$

$$D_t^\alpha[cf(t)]=cD_t^\alpha f(t) \quad (5)$$

2. Methodology

For a given the following nonlinear FDEs of the type

$$\phi(u_1\dots u_k, D_t^\alpha u_1\dots D_t^\alpha u_k, D_{x_1}^\alpha u_1..D_{x_1}^\alpha u_k\dots,\dots D_{x_n}^\alpha u_1..D_{x_n}^\alpha u_k, D_t^{2\alpha}u_1\dots D_t^{2\alpha}u_1\dots D_t^{2\alpha}u_k.D_t^{2\alpha}u_k.D_{x_1}^{2\alpha}u_1)=0, 0<\alpha<1. \quad (6)$$

where $u_i(t,x_1,\dots,x_n)$ is an unknown function, $(i=1\dots,k)$ and ϕ is a polynomial of u and its partial fractional derivatives, in which the highest order derivatives and the nonlinear terms are involved

Step. 1. Suppose that

$$u_i(t,x_1,x_2,\dots,x_n)=U_i(\xi),$$
$$\xi=ct+k_1x_1+k_2x_2+\dots+k_nx_n+\xi_0 \quad (7)$$

Then Eq. (6) reduce to

$$\psi(U_1...U_k, c^\alpha D_\xi^\alpha U_1\dots c^\alpha D_\xi^\alpha U_k, k_1^\alpha D_\xi^\alpha U_1\dots k_1^\alpha D_\xi^\alpha U_k.., k_n^\alpha D_\xi^\alpha U_1\dots)=0, 0<\alpha<1. \quad (8)$$

Step. 2. The next crucial step is that we are looking for the solution of Eq. (8) as

$$U_i(\xi)=\sum_{i=0}^{M}a_i\left(\frac{D_\xi^\alpha G(\xi)}{G(\xi)}\right)^i, i=1,2,\dots, \quad (9)$$

where $a_i(i=0,1,2,\dots)$ are constants to be determined later,the positive integer M can be determined by considering the homogeneous balance between the highest order derivative and nonlinear terms appearing in (8), $G=G(\xi)$ satisfies Eq. (1). Inserting Eq. (9) into (8) and using Eq. (1), collecting all terms with the same order $\left(\frac{D_\xi^\alpha G(\xi)}{G(\xi)}\right)$ together, the left hand side of Eq. (8) is converted into another polynomial in $\left(\frac{D_\xi^\alpha G(\xi)}{G(\xi)}\right)$. Equating each coefficients of this polynomial to zero, we obtain a set of algebraic equations for a_i, $(i=0,1,2,\dots)$, k and c.

Step. 3. Solving the equations system in step 2, and using the general solutions of Eq. (1) [27,28], we can construct a variety of solutions for Eq. (6).

3. New applications

To illustrate the effectiveness and the advantages of the proposed method, we consider two models of special interest of fractional space–time, namely, the fractional KdV equation on the dust ion acoustic (DIA) waves in dusty plasma and the space–time Boussinesq fractional equation.

Example 3.1. The space–time fractional modified KdV equation

Let us first consider space–time fractional modified KdV equation [26]

$$D_t^\alpha\phi(x,t)+\nu\phi^{1/2}(x,t)D_x^\beta\phi(x,t)+\delta D_x^{\beta\beta\beta}\phi(x,t)=0, 0<\alpha<1, 0<\beta<1 \quad (10)$$

where x and t are space and time variables, respectively. The coefficients ν and δ are defined as

$$\nu=\frac{2b}{\lambda(1+\nu)}, \delta=\frac{1}{\lambda^2(1+\nu)}$$

Eq. (10) is called the space–time fractional KdV that descibes the nonlinear propagation of DIA solitary wave. To begin with, suppose that $\alpha=\beta$, $\phi(x,t)=U^2(\xi)$, where $\xi=kx+ct+\xi_0$, k, c, ξ_0 are all constants to be determined later with k, c, m $\neq 0$. Then by using of Eq. (5), we have

$$D_x^\alpha U(\xi)=(D_\xi^\alpha U)(\xi_x^{'})^\alpha=k^\alpha D_\xi^\alpha U,$$

$$D_t^\alpha U(\xi)=(D_\xi^\alpha U)(\xi_t^{'})^\alpha=c^\alpha D_\xi^\alpha U$$

Then Eq. (10) reduces to

$$c^\alpha U(\xi)D_\xi^\alpha U(\xi)+\delta k^\alpha U^2(\xi)D_\xi^\alpha U(\xi)+\nu k^{3\alpha}[U(\xi)D_\xi^{\alpha\alpha\alpha}U(\xi)+3D_\xi^\alpha U(\xi)D_\xi^{\alpha\alpha}U(\xi)]=0 \quad (11)$$

Suppose that the solution of Eq. (11) can be expressed by

$$U(\xi)=\sum_{i=0}^{M}a_i\left(\frac{D_\xi^\alpha G(\xi)}{G(\xi)}\right)^i, \quad (12)$$

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