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Original Article

On controlled propagation of nonautonomous dispersive coupled Whitham–Broer–Kaup equation in shallow water

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Abstract

In this paper, the dispersive coupled Whitham–Broer–Kaup (DCWBK) equation with time-dependent coefficients describing the propagation of the shallow water waves are obtained. The propagation of solitons and elliptic (or chirped) waves can be manipulated by suitable variations of the dispersion coefficient. Here, controllable transmission of the surface waves for soliton similariton pairs with the snoidal backgrounds is considered. It is found that, when the dispersion coefficient is taken as increasing, the velocity is increasing with the dispersion coefficient increasing. While this holds vice versa for the height of propagation wave.

Keywords: Dispersive coupled Whitham–Broer–Kaup (DCWBK) equation; Unified method (UM); Dispersion coefficient; Controlled.

1. Introduction

In recent years, the propagation of long surface water waves has many applications in the phenomena of physical and engineering. Many completely integral models have been presented in bi-directional surface wave propagation, when the depth is small relative to the scale of the waves propagating, such as Boussinesq-types equations [1,2], Broer–Kaup system [3,4], Boussinesq–Burgers equations [5], The Kadomtsev–Petviashvili (KP) equation [6] and the Camassa–Holm (CH) equation [7].

Since the nonlinear evolution equations (NLEEs) with time dependent, are called the nonautonomous systems [8–10]. The features dynamical of soliton from those systems are capable of all properties for a long distances with the negligible weaklings [11].

Several kinds of methods in the literatures have been developed to construct exact solutions to nonlinear differential equations are considered in [12–26].

The dispersive long (surface) water waves with time-dependent coefficient are given by the equations:

$$u_t - u\,u_x + v_x + \beta(t)\,u_{xxx} = 0\,,$$
$$v_t + (u\,v)_x + \alpha(t)\,u_{xxx} - \beta(t)\,v_{xxx} = 0\,, \qquad (1)$$

where $u(x, t)$ is the field of horizontal velocity, $v(x, t)$ is the height waves, $\alpha(t)$ and $\beta(t)$ are the dispersion coefficients varying. If $\alpha(t)$ and $\beta(t)$ are real constant and with represent different diffusion powers (u_{xx} and v_{xx}), Eq. (1) reduces to coupled Whitham–Broer–Kaup (CWBK) equation [27–33]. If $\alpha(t) = 1$ and $\beta(t) = 0$, Eq. (1) becomes the modified Boussinesq equations [34,35].

In the present paper, we will search to find soliton and other exact solutions of the following Eq. (1) by using the unified method (UM) [36,37]. The dispersion coefficients are demonstrated to investigate the different geometrical structures for polynomial and rational solutions.

Now, we consider the coupled evolution equations:

$$F_k(x,\ t,\ u_{i_1}, u_{i_2}, \dots u_{i_1 t}, u_{i_2 t} \dots) = 0,\ k,\ i_j, = 1, 2 \dots s \qquad (2)$$

where F_k are polynomials in their argument. When x, y and t are missing in Eq. (1), then it has traveling wave solutions

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(TWS) for single (TWS), we have

$$G_k(U_{i_1}, U_{i_2}, \dots U'_{i_1}, U'_{i_2}, \dots U''_{i_1}, U''_{i_2} \dots) = 0, k,\ i_j = 1, 2, \dots s$$
$$U' = \frac{dU_i}{d\eta},\ \eta = \kappa\, x + \int \omega(t)\, dt. \tag{3}$$

This paper is organized as follows. In Section 2, the unified method is considered. Section 3 is devoted to find the explicit general wave solutions. The main results are illustrated via the solutions with figures. Finally, in Section 4, some conclusions are given.

2. The unified method

In the present paper, we use the unified method (UM) [36,37]. This method is classified to be polynomials or rational functions solutions, which follow as :

To search for solutions of Eq. (1), the unified method suggests that the solution is given by

2.1. Polynomials solutions

$$u_j(\eta, t) = \sum_{i=0}^{n_j} a_{i_j}(t)\, \varphi^i(\eta),\ j = 1, 2, \dots s$$
$$(\varphi'(\eta, t))^{pk} = \sum_{j=0}^{j=pk} c_j(t)\, \varphi^j(\eta),\ p = 1, 2, \tag{4}$$

where $\varphi(\eta)$ is the auxiliary function, and $a_{i_j}(t)$, $c_j(t)$ are unknown parameters.

In Eq. (1) for n_j and k is determined from the leading analysis and in this case the balance condition in the first and second Eq. (1) is read $n_1 = n_2 = 2(k-1)$. The consistency condition relates the number of equations obtained by substituting Eq. (4) into (1), (namely n_{2_j}), the number of free parameters in polynomial and auxiliary functions (namely n_{1_j}) and the integrability property of Eq. (3). As Eq. (1) is completely integrable, then consistency condition reads $n_{2_j} - n_{1_j} \le m_j$. We mention that, when $p = 1$, the solution of the auxiliary equation gives rise to (explicit or implicit) solutions in elementary functions. While when $p = 2$ they give rise to explicit solutions in Jacobi-elliptic or periodic.

2.2. Rational solutions

To find the rational function solutions of Eq. (3) we assume that

$$u_j(\eta, t) = \sum_{i=0}^{n} p_{i_j}(t)\, \varphi^i(\eta) / \sum_{i=0}^{r} q_{i_j}(t)\, \varphi^i(\eta),\ j = 1, 2, \dots s$$
$$\left(\varphi'(\eta, t)\right)^{pk} = \sum_{i=0}^{i=pk} c_i(t)\, \varphi^i(\eta),\ p = 1, 2, \tag{5}$$

where $p_{i_j}(t)$, $q_{i_j}(t)$ are arbitrary parameters, n, r and k are determined from the leading analysis. It is worth to mention that the balance conditions in this case be obtained as in the case of polynomial solutions but n is replaced by $n - r$. Here again, the condition for the existence of the solutions in Eq. (3) is determined from the consistency equation. Indeed, when $k = 1$ in the solution of the second equation in (5) was suggested to describe "a jet stream" or (wave pattern).

Steps of computation:

When substituting from Eq. (4) (or (5)) into Eq. (3), we get the principle equations and the following steps are done.

1-Solve the principle equations.

2-Solve the auxiliary equations.

3-Find the exact solutions.

4-Check that the solutions obtained satisfies Eq. (3).

3. Solutions of Eq. (1)

The objective of this section is to construct the exact traveling wave solutions (TWS) of Eq. (1) by using the (UM). Under the varying dispersion is bounded functions, the propagation of solitary, soliton, elliptical, chirped and periodic waves are obtained.

Let us, we use the transformations $u(x,t) = U(\eta, t)$, $v(x,t) = V(\eta, t)$ and $\eta = \kappa\, x + \int \omega(t)\, dt$, where κ^{-1} and $\omega(t)$ are designate the characteristic wave lengths and frequency.

Thus Eq. (1) reduce to

$$\omega(t)U' + \kappa(UU' + V') + \kappa^3 \beta(t) U^{(3)} = 0\,,$$
$$\omega(t)V' + \kappa(VU)' + \kappa^3\left(\alpha(t)U^{(3)} - \beta(t)V^{(3)}\right) = 0\,, \tag{6}$$

Here, we use the (UM) to find the polynomial and rational solutions:

(i) Propagation of solitary wave

When $p = 1$, $n_1 = n_2 = 2$ and $k = 2$, we obtain the solitary wave solution in polynomial function solution.

In this case, we take Eq. (5)

$$U(\eta, t) = b_2(t)\, \varphi^2(\eta) + a_1(t)\, \varphi(\eta) + a_0(t)\,,$$
$$V(\eta, t) = b_2(t)\, \varphi^2(\eta) + b_1(t)\, \varphi(\eta) + b_0(t)\,,$$
$$\varphi_\eta = c_2(t)\, \varphi^2(\eta) + c_1(t)\, \varphi(\eta) + c_0(t)\,, \tag{7}$$

where Eq. $(7)_3$ is the auxiliary equation and $a_j(t), b_j(t)$ and $c_i(t)$, $i = 0, 1, 2$ are arbitrary parameters.

From Eq. (7) into Eq. (6), the solutions of Eq. (1) are given by

$$u(x,t) = \frac{-[\kappa\alpha(t) + 3\beta(t)\left(\kappa^3\beta(t)\lambda(t)^2\left(1 - 3\mathrm{sech}^2\left(\tfrac{1}{2}\lambda(t)(\int \omega(t)\, dt + \kappa x)\right)\right) + \omega(t)\right)]}{3\kappa\beta(t)}\,,$$
$$v(x,t) = \frac{\alpha(t)\left(\alpha(t) + 3\kappa^2\beta(t)^2\lambda(t)^2\left(3\mathrm{sech}^2\left(\tfrac{1}{2}\lambda(t)(\int \omega(t)\, dt + \kappa x)\right) - 1\right)\right)}{9\beta(t)^2}\,, \tag{8}$$

where

$$a_0(t) = \frac{-[\,3\beta(t)\left(\kappa^3\left(c_1(t)^2 + 8c_0(t)c_2(t)\right)\beta(t) + \omega(t)\right) + \kappa\alpha(t)]}{3\kappa\beta(t)},$$
$$a_1(t) = -12\kappa^2 c_1(t) c_2(t) \beta(t)\,, \tag{9}$$