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Original Article

Approximation of surface–groundwater interaction mediated by vertical streambank in sloping terrains

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Abstract

New analytical solutions are derived to estimate the interaction of surface and groundwater in a stream–aquifer system. The analytical model consists of an unconfined sloping aquifer of semi-infinite extant, interacting with a stream of varying water level in the presence of a thin vertical sedimentary layer of lesser hydraulic conductivity. Flow of subsurface seepage is characterized by a nonlinear Boussinesq equation subjected to mixed boundary conditions, including a nonlinear Cauchy boundary condition to approximate the flow through the sedimentary layer. Closed form analytical expressions for water head, discharge rate and volumetric exchange are derived by solving the linearized Boussinesq equation using Laplace transform technique. Asymptotic cases such as zero slope, absence of vertical clogging layer and abrupt change in stream-stage can be derived from the main results by taming one or more parameters. Analytical solutions of the linearized Boussinesq equation are compared with numerical solution of corresponding nonlinear equation to assess the validity of the linearization. Advantages of using a nonlinear Robin boundary condition, and combined effects of aquifer parameters on the bank storage characteristic of the aquifer are illustrated with a numerical example.

Keywords: Stream–aquifer interaction; Streambank; Sloping aquifer; Boussinesq equation; Laplace transform.

1. Introduction

Mathematical formulation of subsurface seepage flow over horizontal beds is derived using conventional Dupuit–Forchheimer assumption in which streamlines are considered to be approximately horizontal and the gradient of hydraulic potential same as the absolute slope of the water table [18]. When the bed of an unconfined aquifer bed is sloping, the flow is constrained to the direction nearly parallel to the bed [10,13], i.e. streamlines are approximately parallel to the sloping bed, and equipotential are perpendicular to the bed. The consideration that the flow pattern is controlled by the bed slope yields the discharge rate, q, per unit width of the aquifer by the relation [12]:

$$q = -K\, h \cos^2\beta \left[\frac{\partial h}{\partial x} - \tan\beta\right] \tag{1}$$

where h is the height of the water table measured in the vertical direction from the sloping base; x is spatial coordinate; K is the hydraulic conductivity of the aquifer, and $\tan\beta$ is the bed slope. Considering a representative elementary volume and applying principle of mass balance on it, the flow of subsurface seepage in horizontal and vertical axes system is approximated by the following nonlinear Boussinesq equation:

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) - \tan\beta\frac{\partial h}{\partial x} = \frac{S}{K\cos^2\beta}\frac{\partial h}{\partial t} \tag{2}$$

where S is the specific yield of the aquifer. Eq. (2) is indeed a nonlinear advection–diffusion equation which plays key role in approximation of unconfined groundwater flow over sloping bed. Numerical and experimental studies [19] indicate that the results obtained from Eq. (2) are reasonably valid for bed slope up to 30%, i.e. $\tan\beta \le 0.3$. Due to nonlinearity, Eq. (2) is analytically intractable. However, approximate analytical solutions of Boussinesq equation have been used in numerous studies to analyze the transient behavior of

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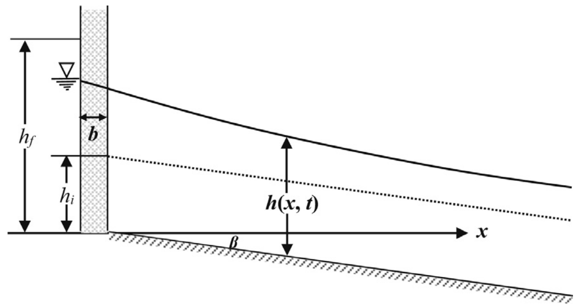

Fig. 1. Schematic diagram of the stream–aquifer system consisting of an unconfined aquifer of semi-infinite extent overlying an impervious sloping base and interacting with a time-varying stream in presence of thin vertical sedimentary layer.

stream–aquifer models under varying hydrological conditions [1–4,11,15,5–7,23,8].

Most of the fundamental studies on surface–groundwater interactions focus on estimation of water exchange between streams and aquifers, without taking the effects of vertical clogging layer into account [14,16,22,24]. Often, if not always, the water interaction between streams and aquifers is mediated by a vertical layer of low permeable sedimentary deposits, henceforth referred to as streambank. Such layers are formed by deposition of silt particles or loading of waste along river/stream bank in rivers whose streamflow is sluggish [9,20,25]. These layers decelerate the seepage flow from river to aquifer and vice-versa. For horizontal strata, the approach to estimate the flow through the streambank is to consider the seepage velocity a function of difference between stream stage and aquifer's water table [17]; however, there is not much discussion in the literature on approximation of seepage flow through streambank when a fully penetrating stream interacts with an unconfined sloping aquifer.

Recently, Bansal et al. [8] derived an appropriate form of boundary condition to simulate the flow through streambank in an archetypical stream–aquifer model in which the aquifer is resting on sloping impervious bed. The problem of seepage flow between an unconfined aquifer of semi-infinite extent and a fully penetrating stream of time-varying water level in the presence of vertical streambank (see Fig. 1) is characterized by the following linearized form of Boussinesq Eq. (2).

$$\frac{\partial^2 h}{\partial x^2} - \frac{\tan\beta}{h_{\text{avg}}}\frac{\partial h}{\partial x} = \frac{S}{K\, h_{\text{avg}}\cos^2\beta}\frac{\partial h}{\partial t} \tag{3}$$

where h_{avg} is the average saturated depth of the aquifer given by an iterative formula $h_{\text{avg}} = (h_i + h_t)/2$; h_i is the initial depth and h_t is the variable height at time t, at the end of which h_{avg} is calculated. Considering that the flow at stream–aquifer interface is made up of two terms, namely flow due to head variation across the streambank, and flow due to bed slope, they derived an appropriate nonlinear boundary condition at the stream–aquifer interface, given by

$$-K\cos^2\beta\, h\left(x=0^+,\, t\right)\left(\frac{\partial h}{\partial x}\right)_{x=0} = -k\frac{h^2\left(x=0^+,\, t\right) - h_s^2(t)}{2b} \tag{4}$$

where k and b are the hydraulic conductivity and thickness of the clogging layer. The stream-stage variation $h_s(t)$ from an initial level h_i to a final level h_f, controlled by a nonnegative parameter λ, is given by the transient relation

$$h_s(t) = h_f - \left(h_f - h_i\right)e^{-\lambda t} \tag{5}$$

Moreover, an initial condition of depth independent water table reads

$$h(x,\, t=0) = h_i \tag{6}$$

and a far-end boundary condition is

$$h(x \to \infty,\, t) = h_i \tag{7}$$

Assuming that the head loss across streambank is small, Bansal et al. [8] used the approximation $|h_s(t) - h(x=0^+, t)| << h(x=0^+, t)$. With this consideration, they simulated the nonlinear boundary condition (4) by a simpler linear equation

$$-K\cos^2\beta\left(\frac{\partial h}{\partial x}\right)_{x=0} = -k\frac{h\left(x=0^+,\, t\right) - h_s(t)}{b} \tag{8}$$

The term $(Kb\cos^2\beta)/k = r$ denotes the streambed retardation coefficient or streambank leakance in sloping aquifer. This parameter controls the degree of hydraulic connection between the stream and aquifer, and depends on the physical properties of the sediments that make this layer. It is demonstrated in Bansal et al. [8] that the linearization of boundary condition (4) yields acceptable results only when the stream-stage variations and streambank leakance are in narrow range ($h_f - h_i < 30\%$ of h_i and streambank leakance $r < 10$). When the ratios $|h_f - h_i|/h_i$ and streambank leakance are large, the analytical solutions developed by Bansal et al. [8] varied considerably from the numerical solution of the corresponding to nonlinear boundary condition (4). These facts highlight practical limitations of the results developed by Bansal et al. [8].

The aim of the present study is to develop new analytical solution of the linearized Boussinesq equation subjected to nonlinear boundary condition (4) along with initial condition (6) and a far-end boundary condition (7). Closed form analytical expressions are obtained using Laplace transform method to predict the spatio-temporal distribution of water head, rate of discharge and volumetric exchange between stream and aquifer for rise and decline in the stream-stage. Few previously known results are derived as special cases of the main results of this study. Numerical solution of the nonlinear Boussinesq Eq. (2) subjected to nonlinear condition (4) are also developed using an explicit Dufort–Frankel method to assess the validity of analytical solutions for large value of the ratios $|h_f - h_i|/h_i$ and streambank leakance ratio r.

2. Analytical solution of linearized Boussinesq equation

Assuming that the unconfined downward sloping aquifer is homogeneous and isotropic, and the streamlines are nearly parallel to the sloping bed, the flow of subsurface seepage is governed by nonlinear Boussinesq Eq. (2). This equation is a second order parabolic partial differential equation whose