Available online at www.sciencedirect.com
ScienceDirect

Journal of Ocean Engineering and Science 000 (2016) 1–7
www.elsevier.com/locate/joes

Original Article

Two efficient reliable methods for solving fractional fifth order modified Sawada–Kotera equation appearing in mathematical physics

S. Saha Ray*, S. Sahoo

National Institute of Technology, Department of Mathematics, Rourkela 769008, India

Received 4 June 2016; accepted 27 June 2016
Available online xxx

Abstract

The present paper deals with two reliable efficient methods viz. tanh-sech method and modified Kudryashov method, which are used to solve time-fractional nonlinear evolution equation. For delineating the legitimacy of proposed methods, we employ it to the time-fractional fifth-order modified Sawada–Kotera equations. As a consequence, we effectively obtained more new exact solutions for time-fractional fifth-order modified Sawada–Kotera equation. We have also presented the numerical simulations for time-fractional fifth-order modified Sawada–Kotera equation by means of three dimensional plots.

Keywords: Time-fractional fifth-order modified Sawada–Kotera equation; Local fractional calculus; Tanh-sech method; Modified Kudryashov method.

1. Introduction

Fractional calculus is a wide area of mathematics which offers derivatives and integrals of arbitrary orders. In the last few years, fractional calculus [1,2] has been broadly investigated due to their large functions in mathematics, physics and engineering similar to viscoelasticity, signal processing, electromagnetism, fluid mechanics, electrochemistry and so forth. Fractional differential equations are extensively used in modeling of physical phenomena in various fields of science and engineering. For this reason, we need to develop new reliable and efficient process for the solution of fractional differential equations.

We have considered here the time-fractional fifth-order modified Sawada–Kotera (mS–K) equation [3,4]

$$D_t^{\alpha} u - u_{xxxxx} + \left(5u_x u_{xx} + 5uu_x^2 + 5u^2 u_{xx} - u^5\right)_x = 0, \quad 0 < \alpha \le 1, \tag{1.1}$$

which is directly derived from the time-fractional fifth-order Sawada–Kotera (S–K) equation [5–7]

$$D_t^{\alpha} u + 5u^2 u_x + 5u_x u_{xx} + 5uu_{xxx} + u_{xxxxx} = 0, \quad 0 < \alpha \le 1,$$

and time-fractional fifth-order Kaup–Kupershmidt (K–K) equation [8–10]

$$D_t^{\alpha} u + 45u^2 u_x - 15pu_x u_{xx} - 15uu_{xxx} + u_{xxxxx} = 0, \quad 0 < \alpha \le 1,$$

by applying the Miura transformations. The time-fractional fifth-order modified Sawada–Kotera equation has the combined physical nature of both fractional S–K and K–K equations.

The present paper is dedicated to build the exact solitary solutions for time-fractional fifth-order mS–K equation utilising fairly new techniques, in particular the tanh-sech method [5,11–13] and modified Kudryashov method [14,15]. To the best of knowledge of authors, no prior exploration work has been carried out using proposed methods for solving time-fractional fifth-order modified Sawada–Kotera equation.

The remainder of this paper is organised as follows: definitions with properties of local fractional calculus [16–18] have been discussed in Section 2. We explained the algorithm of proposed tanh-sech method and Kudryashov method in Section 3. In Section 4, the exact solitary wave solutions for the time-fractional fifth order mS–K equation are derived. In Section 5, the natures of the solutions are examined by means of numerical simulation. The present paper concluded with Section 6.

* Corresponding author.
E-mail address: santanusaharay@yahoo.com (S. Saha Ray).

http://dx.doi.org/10.1016/j.joes.2016.06.002
2468-0133/

2. Local fractional derivative and its properties

Definition 2.1. Let $g(t) \in C_\alpha(a,b)$. Local fractional derivative of $g(t)$ of order α at $t = t_0$ is defined as [16–18]

$$g^{(\alpha)}(t_0) = \left.\frac{d^\alpha g(t)}{dt^\alpha}\right|_{t=t_0} = \lim_{t\to t_0} \frac{\Delta^\alpha (g(t) - g(t_0))}{(t-t_0)^\alpha}, \tag{2.1.1}$$

where $\Delta^\alpha (g(t) - g(t_0)) \cong \Gamma(1+\alpha)(g(t) - g(t_0))$ and $0 < \alpha \le 1$.

Properties of local fractional derivative [16–19]

I. If $y(t) = (g \circ u)(t)$ where $u(t) = h(t)$, then we have

$$\frac{d^\alpha y(t)}{dt^\alpha} = g^{(\alpha)}(h(t))\left(h^{(1)}(t)\right)^\alpha, \tag{2.1.2}$$

when $g^{(\alpha)}(h(t))$ and $h^{(1)}(t)$ exist.

II. If $y(t) = (g \circ u)(t)$ where $u(t) = h(t)$, then we have

$$\frac{d^\alpha y(t)}{dt^\alpha} = g^{(1)}(h(t))h^{(\alpha)}(t), \tag{2.1.3}$$

when $g^{(1)}(h(t))$ and $h^{(\alpha)}(t)$ exist.

3. Alogorithm of proposed methods

3.1. Brief description of algorithm for the new proposed tanh-sech method

In this section, the tanh-sech method [5, 11–13] has been used to find the exact solutions of Eq. (1.1). The fundamental steps of the proposed method are illustrated as follows:

Step 1: We consider the most general form of nonlinear fractional partial differential equations with two independent variables x and t given by

$$G(u, u_x, u_{xx}, u_{xxx}, \ldots D_t^\alpha u, \ldots) = 0, \quad 0 < \alpha \le 1 \tag{3.1.1}$$

where G is a polynomial in $u(x,t)$. It may be noted that in Eq. (3.1.1) some nonlinear term with higher order partial derivatives are included.

Step 2: Let

$$u(x,t) = \Phi(\xi), \quad \xi = c\left(x - \frac{vt^\alpha}{\Gamma(\alpha+1)}\right) \tag{3.1.2}$$

be fractional complex transform [20–22], which can be used for reducing Eq. (3.1.1) into nonlinear ordinary differential equation. Here c and v are arbitrary constants.

By using the chain rule Eq. (2.1.3) [20,21], we have

$$D_t^\alpha u = \sigma_t \Phi_\xi D_t^\alpha \xi,$$

where σ_t is the fractal indexes [21,22]. Let us assume without loss of generality that $\sigma_t = k$, where k is a arbitrary constant.

By applying Eq. (3.1.2) the Eq. (3.1.1) can be written as

$$G(\Phi, c\Phi', c^2\Phi'', c^3\Phi''', \ldots, -cv\Phi', \ldots) = 0. \tag{3.1.3}$$

Step 3: By tanh-sech method, the solution of Eq. (3.1.3) can be written as follows:

$$\Phi(\xi) = a_0 + \sum_{i=1}^{n} a_i Y^i. \tag{3.1.4}$$

Using homogenous balancing principle, equating nonlinear term and highest order derivative term of Eq. (3.1.3), the value of n can be determined.

In this method, we take tanh-sech method, let $Y = \tanh(\xi)$. Using chain rule, we obtain the derivatives of $\Phi(\xi)$, which are given as follows:

$$\frac{d\Phi}{d\xi} \to \left(1-Y^2\right)\frac{d\Phi}{dY},$$
$$\frac{d^2\Phi}{d\xi^2} \to \left(1-Y^2\right)\left(-2Y\frac{d\Phi}{dY} + \left(1-Y^2\right)\frac{d^2\Phi}{dY^2}\right). \tag{3.1.5}$$

Similarly, the higher-order derivatives can be found.

Step 4: Then by substituting Eq. (3.1.4) into Eq. (3.1.3) and making use of Eq. (3.1.5) followed by collecting all terms with the same degree of Y^i $(i = 0, 1, 2, \ldots)$ together, the Eq. (3.1.3) is transformed into an another polynomial in Y^i $(i = 0, 1, 2, \ldots)$. Equating every coefficient of this polynomial to zero, we will get a set of algebraic equations for a_i $(i = 0, 1, 2, \ldots, n)$, v and c.

Step 5: Solving the obtained algebraic systems in Step 4 and in the same time substituting these constants a_i $(i = 0, 1, 2, \ldots, n)$, v and c into Eq. (3.1.4), we can get the explicit solutions of Eq. (3.1.1) immediately.

3.2. Brief description of algorithm for the new proposed modified Kudryashov method

In this section, the modified Kudryashov method [14,15] has been used to find the exact solutions of Eq. (1.1). The fundamental steps of the proposed method are illustrated as follows:

Step 1: We consider the most general form of nonlinear fractional partial differential equations with two independent variables x and t given by

$$H(u, u_x, u_{xx}, u_{xxx}, \ldots D_t^\alpha u, \ldots) = 0, \quad 0 < \alpha \le 1, \tag{3.2.1}$$

where H is a polynomial in $u(x,t)$. It may be noted that in Eq. (3.2.1) some nonlinear term with higher order partial derivatives are included.

Step 2: Let

$$u(x,t) = \Psi(\zeta), \quad \xi = lx + \frac{\gamma t^\alpha}{\Gamma(\alpha+1)}. \tag{3.2.2}$$

be fractional complex transform [20–22], which can be used for reducing Eq. (3.2.1) into nonlinear ordinary differential equation. Here l and γ are arbitrary constants.