# FAULT MODEL OF AN INVERTER-FED PM MOTOR FOR X-BY-WIRE SYSTEMS

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Abstract — A dynamic model of an x-by-wire actuator applicable in e.g. Electronic Power Steering, Brake-by-Wire or Electronic Parking Brake is presented. The actuator includes Permanent Magnet Synchronous Machine (PMSM), inverter, controller and a simplified mechanical load model. The PMSM is approximated by a set of differential equations describing each coil or each phase, with the possibility to simulate winding short circuit faults. A three-leg inverter model is designed, where all transistors and diodes inside may have separate short circuit or open circuit condition to simulate inverter faults. A field-oriented torque and speed cascade controller is implemented. In "one fault at one time" failure condition, simulation results from model with independent coil equations and independent phase equations are investigated. 

Keywords - permanent magnet motors, stator windings, inverters, pulse width modulation, dynamic models, fault, cascade control, closed-loop, open-loop, automobiles

## 1. INTRODUCTION

The control of the motion of cars may be accomplished in the future increasingly by x-by-wire actuators. In case of brake systems for example the necessary forces on the car´s wheel could be created electromechanically instead of hydraulically, which is commonly used nowadays. In case of "steer-by-wire" system there is a better possibility to stabilize the steering of the car in dangerous situations like slippery road with an autonomous steering control. Fault-tolerance is a very important feature of such a system, as failure of one part must not lead to malfunction, thus keeping up safe operational performance in any case.

The drive system, which is investigated here for x-by-wire, consists of a MOSFET Pulse Width Modulation (PWM) inverter, feeding the windings of a Permanent Magnet Synchronous Machine (PMSM), which is acting via gear, e.g. on the wheel position system in case of use as "steer-by-wire" drive. To study in different fault situations the behaviour of the drive system and optimize the fault tolerant performance, the dynamic model of the electrical and mechanical system is needed. A PMSM with tooth-coil winding is used as the motor, because of its high efficiency and good dynamic capability. This paper introduces a dynamic model of the x-by-wire system established in MATLAB/Simulink, which includes the machine, the inverter, a cascade controller and a simplified mechanical model. The machine model is described with independent coil equations or three independent phase equations. Short circuit faults of different numbers of coils in a phase can be simulated. The torsion resonance of the elastic coupling between the motor and the driven load is considered. The three phase inverter model, operated from a DC voltage source (e.g. a battery), consists of three modules. Each module represents one inverter leg. The diodes and the transistors in one leg may be set by flags to be short circuit or open circuit to simulate different inverter faults. The torque and speed cascade controller parameters are optimized by the amplitude optimum and symmetrical optimum. Different kinds of coil short circuit and inverter faults are simulated in the model with and without speed controlled operation. In a real x-by-wire system in most cases the motor just rotates several radians at each actuator motion, while in the simulation it is assumed that it rotates with the mechanical speed of 375 rpm at steady state (the electrical speed is 50 Hz). This condition shall be used later on in a test-rig setup. When the controller is not applied, a *U/f*-ramp is used to start the PMSM. The deformations of time responses of the currents, the electromagnetic torque and the mechanical speed of the machine are investigated and analysed at faulty condition. "One fault at one time" is considered here.

## 2. MATHEMATICAL MODEL OF THE PMSM

For symmetrical synchronous machines, the *d-q* reference frame equations are used to build the PMSM model. In the case of winding fault ($m$ = 3), the three phases are not symmetrical.

Winding model 1: The equations of independent coils of the machine are derived for this purpose as a numerical model considering the mutual flux linkage between stator coils and between stator and rotor by coil inductances. The flux in each coil includes flux produced by the coil current itself, by the other coil currents, and by the rotor permanent magnets. Here a $2p$ = 16 pole tooth-coil winding with $q$ = 3/8 slots per pole and phase is investigated, so $2pqm$ = 18 coils are considered. The induced voltage in the $i$-th coil ($i$ = 1, 2, …18) is: (see *Appendix 2*: List of symbols)

$$u_{c,i} = -c_{s,i} k_p \hat{\Psi}_{pc} \cdot p\Omega_m \cdot \sin\left[p\Omega_m t + p\gamma_0 - (i-1)p\alpha_Q\right] + \left(L_{ch} + L_{cQ} + L_{cb}\right)\frac{di_{c,i}}{dt}$$
$$+\left(-c_{s,i}\right)M_{ch}\sum_{\substack{j=1,\dots 18\\ j\neq i}} c_{s,j}\frac{di_{c,j}}{dt} + \left(-c_{s,i}\right)M_{cQ}\left(c_{s,i-1}\frac{di_{c,i-1}}{dt} + c_{s,i+1}\frac{di_{c,i+1}}{dt}\right) + R_c i_{c,i} \quad (1)$$

The electrical torque is calculated from internal power:

$$M_e = \sum_{i=1,\dots 18}\left(-c_{s,i} k_p \hat{\Psi}_{pc} \cdot p \cdot \sin\left[p\Omega_m t + p\gamma_0 - (i-1)p\alpha_Q\right] \cdot i_{c,i}\right) \quad (2)$$

where $c_{si} = \pm 1$ gives the winding sense of each coil (Table 1). Rotor space harmonics are neglected, so only fundamental rotor flux linkage is considered, leading to fundamental pitching factor $k_p$ per coil. The equations of the healthy star-connected PMSM are derived for given potentials $v_U$, $v_V$, $v_W$ at terminal and $v_N$ at star point:

$$v_U - v_N - u_{c,1} - u_{c,2} - u_{c,3} - u_{c,10} - u_{c,11} - u_{c,12} = 0 \quad (3)$$
$$v_V - v_N - -u_{c,4} - u_{c,5} - u_{c,6} - u_{c,13} - u_{c,14} - u_{c,15} = 0 \quad (4)$$
$$v_W - v_N - -u_{c,7} - u_{c,8} - u_{c,9} - u_{c,16} - u_{c,17} - u_{c,18} = 0 \quad (5)$$
$$i_U + i_V + i_W = 0 \quad (6)$$

Note, that coil currents are

$$i_{c,1} = i_{c,2} = i_{c,3} = i_{c,10} = i_{c,11} = i_{c,12} = i_U \quad (7)$$

according to Table 1, and accordingly for $i_V$ and $i_W$. This corresponds to series connection of all 6 coils per phase (number of parallel winding branches per phase $a$ = 1).

Considering the torsion resonance of the elastic shaft between the motor and the load, the mechanical system equations (8) and (9) are added:

$$J_M \ddot{\gamma}_m = M_e - c \cdot (\gamma_m - \gamma_L) \quad (8)$$
$$J_L \ddot{\gamma}_L = c \cdot (\gamma_m - \gamma_L) - M_L \quad (9)$$

For stiff shaft ($c \rightarrow \infty$) (8) and (9) simplifies to (10):

$$(J_M + J_L)\ddot{\gamma}_m = M_e - M_L \quad (10)$$

If, for example, in fault case the first $k$ tooth coils in phase U are short circuit, equations are changed:

$$\sum_{i=1,\dots k} u_{c,i} = 0 \quad (11)$$
$$i_{c,1} = i_{c,2} = \dots = i_{c,k} = i_{sc} \quad (12)$$

For simulation, the set of equations (1)-(12) is implemented in MATLAB/Simulink.

Winding model 2: By combining (1), (3), (4) and (5), the equations of the healthy model are containing phase inductances and phase flux linkage:

$$v_U - v_N = \left(L_{hUU} + L_{\sigma U}\right)\frac{di_U}{dt} - M_{UV}\frac{di_V}{dt} - M_{UW}\frac{di_W}{dt} - p\Omega_m \cdot \Psi_{pU} \cdot \sin(p\Omega_m t + p\gamma_0') + R_U I_U \quad (13)$$
$$v_V - v_N = \left(L_{hVV} + L_{\sigma V}\right)\frac{di_V}{dt} - M_{UV}\frac{di_U}{dt} - M_{VW}\frac{di_W}{dt} - p\Omega_m \cdot \Psi_{pV} \cdot \sin(p\Omega_m t + p\gamma_0' - \frac{2\pi}{3}) + R_V I_V \quad (14)$$
$$v_W - v_N = \left(L_{hWW} + L_{\sigma W}\right)\frac{di_W}{dt} - M_{UW}\frac{di_U}{dt} - M_{VW}\frac{di_V}{dt} - p\Omega_m \cdot \Psi_{pW} \cdot \sin(p\Omega_m t + p\gamma_0' - \frac{4\pi}{3}) + R_W I_W \quad (15)$$
$$i_U + i_V + i_W = 0 \quad (16)$$

Simplified consideration of coil short circuits may be done, if phase equation set (13)-(16) is used. In case of e.g. short circuit of coil 1 ($k$ = 1) in phase U, the total number of turns of remaining phase U is $N'$ = $N\cdot(6-1)/6$, where $N = 2pq\cdot N_c/a$ is number of turns per phase in healthy condition. This changes the phase inductances proportional to $(N'/N)^2$. The back EMF, which e.g. in phase U is $p\Omega_m \hat{\Psi}_{pU} \sim k_w \hat{\Psi}_{p,c}$, is changed by a new fundamental winding factor $k_w'$ instead of $k_w$, calculated according to winding scheme of Table 1.

This simplified fault model can not determine the coil short circuit current $i_{sc}$ and is not considering the proper flux linkage of coils under fault condition. It is only a roughly estimate. The asymmetrical equations (13)-(15) yield asymmetrical phase currents, which are used for calculating torque:

$$M_e = -\left[p\Psi_{pU} \cdot i_U \cdot \sin(p\Omega_m t + p\gamma_0') + \Psi_{pV} \cdot i_V \cdot \sin\left(p\Omega_m t + p\gamma_0' - \frac{2\pi}{3}\right)\right.$$
$$\left. + \Psi_{pW} \cdot i_W \cdot \sin\left(p\Omega_m t + p\gamma_0' - \frac{4\pi}{3}\right)\right] \quad (17)$$

The rated data of the test motor are shown in Table 2.

## 3. MODELING OF THE INVERTER

It is assumed that ideal elements (diodes, transistors) are used in the inverter, so the resistances of these elements are zero, when conducting, and infinite,

Table 1 Winding Scheme of the Test Motor

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $c_{s,i}$ | +1 | -1 | +1 | +1 | -1 | +1 | +1 | -1 | +1 |
| phase | +U | -U | +U | +V | -V | +V | +W | -W | +W |

| $i$ | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|
| $c_{s,i}$ | +1 | -1 | +1 | +1 | -1 | +1 | +1 | -1 | +1 |
| phase | +U | -U | +U | +V | -V | +V | +W | -W | +W |

Table 2 Rated Data of the Test Motor

| $2p$ | $U_{N,phase}$ / V | $I_N$ / A | $M_N$ / Nm | $n_N$ / rpm |
|---|---|---|---|---|
| 16 | 8.85, Y | 5 | 1.7 | 375 |