THE WINDUP PROBLEM IN REPETITIVE CONTROL: A SIMPLE ANTI-WINDUP STRATEGY

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Abstract: In this work, we address the problem when the output of a repetitive controller is subjected to actuator saturation. We provide insight into the effects of saturating actuators on the closed-loop performance of a prototype repetitive controller. In order to improve the dynamic transient we propose an anti wind-up strategy tailored to repetitive controllers.

Keywords: actuator saturation, anti-windup, repetitive control.

1. INTRODUCTION

The effects of saturating actuators in control loops are normally manifested as a loss of performance, which can include undesirable transients and even loss of stability. In general terms, it is well known that if a controller has an integrator or unstable modes it will be prone to experiencing saturation (Galttfelder and Schaufelberger, 2003). Once the actuator saturates, the controlled plant behaves as an open loop system, and the controller will continue integrating errors that cannot compensate for, leading to a degradation of the closed loop response. This problem is known as "windup".

Several methods have been proposed to deal with the windup problem in a general framework. Edwards and Postlethwaite (1999) provide a general review of the existing methods for dealing with the windup problem. Most of them have been formulated for continuous time systems, and there are just a few methods dealing with discrete time systems. Park and Choi (1997) proposed a method based on discrete time state space representations. Their method considers the minimization of a cost function in terms of the states in the absence and presence of saturating actuators. More recently, Grimm et al (2003) also describe a formalism, in term of the l_2 norm, based on a state space representation of the responses of the unconstrained linear closed-loop and the anti-windup compensated closed-loop with saturation. On the other hand, Graebe and Ahlén (1996) present a method, based on the work by Rönnbäck (1993), that considers a transfer function description which regards the difference between the saturated and unsaturated outputs as a fictitious disturbance acting on the input of the plant. Their methodology seeks to minimize the transient effect after the control signal desaturation, as measured by the H_2 norm of a linear transfer function, while eliminating the risk of repeated re-saturation and nonlinear oscillations.

In repetitive control, the controller has an internal model of the repetitive signal, which is marginally stable. Also, the repetitive controller involves multiple integrators, each of which is updated in sequence once every repetitive cycle. Therefore, windup is a potential problem in repetitive control when saturating actuators are considered. Notwithstanding the significant research work available on both repetitive control and anti-windup schemes, to the best of our knowledge this is the first attempt to deal in a general framework with the windup problem in a repetitive controller setting. Previous work on this issue can be found in Rönnbäck et al (1993) where an anti-windup strategy for controlling a peristaltic pump with periodic disturbances was described. In addition, Ryu and Longman (1994) describe very briefly an anti-windup strategy for a simple repetitive controller.

Our aim is to characterize a simple anti-windup strategy tailored to repetitive controllers, and shed some light into the influence of the control structure on the response of the controller operating with saturating actuators. We will not design a controller considering the saturating nonlinearity, but follow the retrofit approach instead, i.e., we will add an additional compensator, namely, the anti wind-up compensator, to an existing repetitive controller. We will consider the prototype repetitive controller, even though more general repetitive controllers can also be analyzed using this approach.

This paper is organized as follows: Section 2 describes the basis of repetitive control design, and analyses are carried out when a saturating actuator is in place, between the repetitive controller and the plant. In Section 3 a simple anti-wind up scheme is proposed. Section 4 illustrates by mean of a simple example the main characteristics of the proposed method. Finally, in Section 5 some closing remarks are given.

2. THE REPETITIVE CONTROLLER

Consider a discrete time system described by:

$$A(z^{-1})y(k) = z^{-d}B(z^{-1})[u(k)+d(k)] \quad (1)$$

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} \cdots + b_m z^{-m}$$

where u, y and d are, respectively, the input, output and disturbance signals, z^{-1} represents one step time delay, and d is the pure delay steps. Note that the input-output transfer function is:

$$P(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \quad . \quad (2)$$

Let us also assume that the system is asymptotically stable, i.e., the poles of the transfer function are all inside the unit circle. $B(z^{-1})$ is written as:

$$B(z^{-1}) = B^c\left(z^{-1}\right)B^u\left(z^{-1}\right) \quad (3)$$

$$B^c(z^{-1}) = b_0 + b_1^c z^{-1} + \cdots + b_{mc}^c z^{-mc}$$

$$B^u(z^{-1}) = 1 + b_1^u z^{-1} + \cdots + b_{mu}^u z^{-mu}$$

where $B^c(z^{-1})$ and $B^u(z^{-1})$ contain, respectively, cancelable zeros and uncancelable zeros.

For repetitive (periodic) desired outputs and disturbances with period N, asymptotic regulation may be achieved by a repetitive controller, which is based on the internal model principle (Francis and Wonham, 1975). The feedback controller for (1) needs the internal model of repetitive signals for asymptotic regulation of the error. Such a controller may be represented by

$$U(z) = \frac{k_r z^{-N+d}A(z^{-1})\ B^u(z)}{(1-z^{-N})B^c(z^{-1})b}E(z), \quad (4)$$

$$b > \max_{\omega\varepsilon[0,\pi]} |B^u(e^{j\omega})|^2$$

where

$$B^u(z\ ) = 1 + b_1^u z\ \ + \cdots + b_{mu}^u z^{mu}$$

and $1/(1-z^{-N})$ is the internal model of repetitive signals with period N. The stability condition obtained by Tomizuka et al (1989), i.e., $0 < k_r < 2$, suggests that the repetitive control system described by equation (5) may be robust to parameter variations by selecting the gain k_r to be small. However, it has been observed that the stability of repetitive control systems with the exact internal model of repetitive signals is not robust in the presence of unmodelled dynamics. This problem arises due to the nature of the internal model, i.e., the characteristic roots of the internal model $1/(1-z^{-N})$ are all on the unit circle, which is the stability boundary. This problem may be overcome introducing a low pass filter in the internal model. The repetitive controller with a modified internal model is:

$$G_p(z) = \frac{q(z,z^{-1})\bar{k}_r z^{-N+d}A(z^{-1})B^u(z)}{(1-q(z,z^{-1})z^{-N})B^c(z^{-1})},$$

$$,\bar{k}_r = \frac{k_r}{b},\ b > \max_{\omega\varepsilon[0,\pi]} |B^u(e^{j\omega})|^2 \quad (5)$$

$$q(z,z^{-1}) = \frac{\alpha_1 z^{-1} + \alpha_0 + \alpha_1 z}{\alpha_0 + 2\alpha_1}$$

Notice that $q(z,z^{-1})$ is a low pass filter with zero phase characteristics (Tomizuka, 1987). It can be generalized by introducing higher order terms of z and z^{-1}.

Figure 1 depicts the main components of a repetitive controller and a saturating actuator.

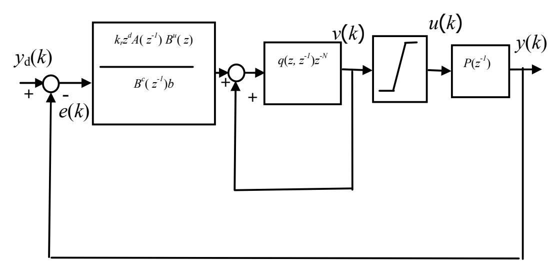

Fig. 1. Repetitive controller with saturating actuator.

From Figure 1, the controller's output in terms of the saturated control output and the reference signal is given by

$$V(z) = -\left[\frac{\bar{k}_r q(z,z^{-1})z^{-N}B^u(z^{-1})B^u(z)}{(1-q(z,z^{-1})z^{-N})}\right]U(z) + \ldots..$$

$$\left[\frac{\bar{k}_r q(z,z^{-1})z^{-N+d}A(z^{-1})B^u(z)}{(1-q(z,z^{-1})z^{-N})B^c(z^{-1})}\right]Y_d(z) \quad (6)$$

In order to analyze the bounded ness of $v(k)$ we will assume a stationary condition, i.e., the reference signal is a bounded periodic signal with period N. Furthermore, the plant dynamics is nominal, i.e. no parameter uncertainties and no ignored dynamics. Under these assumption, if $v(k)$ does not hit the saturation limit during the transient and stationary state, it evolves according to the closed loop dynamics and is bounded.

If $v(k)$ hit the saturation limit during transient, the linear closed loop dynamics no longer apply. The internal model block has always an integrator characteristics, i.e. one pole is at 1. Thus, if $v(k)$ is saturated on either end of the saturation limit, $u(k)$