EFFICIENT IIR NOTCH FILTER DESIGN VIA MULTIRATE FILTERING TARGETED AT HARMONIC DISTURBANCE REJECTION

[*] *Control Systems Technology group*
Technische Universiteit Eindhoven
Eindhoven, The Netherlands

Dennis Bruijnen [*], René van de Molengraft [*] and Maarten Steinbuch [*]

Abstract: In this paper a computationally cheap method is proposed to filter a harmonic series where the first harmonic is a sine at the fundamental frequency f_1 and the super harmonics are sines at frequencies $2f_1, 3f_1, 4f_1$.... The method is based on multirate filter banks. With this method a large amount of IIR notch filters can be implemented much more efficiently than by simply staggering single rate IIR notch filters without deteriorating the frequency range of interest.

Keywords: Notch filters, filter banks, harmonic disturbance rejection

1. INTRODUCTION

Periodic disturbances are often found in practical applications. The field of repetitive control (Hara *et al.*, 1988) deals with control systems subjected to periodic disturbances and exploits the internal model principle (Francis and Wonham, 1975). Also in other research areas, like electric power systems, elimination of harmonics is addressed, (Qidwai and Bettayeb, 1997; Rechka *et al.*, 2003; Czarnecki and III, 2005).

Our interest lies in attenuating harmonic disturbance signals which is a subclass of periodic disturbances. The first harmonic is a sine at frequency f_1, the so-called fundamental frequency, and the super harmonics are sines at the frequencies kf_1 with $k \in \{2, 3, 4, ...\}$. Furthermore, it is assumed that the harmonic disturbances contain higher frequencies than the frequency range of interest. Consider for example control systems where the bandwidth frequency lies in the lower frequency range with respect to the sampling frequency. Above the bandwidth frequency, the only objective is to attenuate disturbances, which does not restrict the sampling rate necessarily. The only requirement is that signals above the bandwidth frequency do not alias such that the signal content below the bandwidth is deteriorated.

To attenuate harmonics at higher frequencies than the frequency range of interest, a lowpass filter would be a natural choice to remove these harmonics. However, the selectivity of such filter is bad. The selectivity is defined as the slope of the magnitude response at the band edge. Signal content in the frequency range of interest will either be deteriorated significantly or the periodic disturbance will not be sufficiently attenuated unless the difference between the frequency range of interest and the disturbance frequency is more than 2-3 decades. The use of notch filters is much more desirable in that case because of their higher selectivity.

Compared to the low pass filter, the filter order will increase dramatically when notches are placed at each harmonic frequency resulting in high computational costs. An alternative which is often used in repetitive control is a FIR comb filter, however, compared to IIR filters, the required number of taps or "order" of the FIR filter is very large to obtain equal selectivity properties, which leads to high computational costs.

In this paper, an efficient method is proposed to implement a large number of IIR notch filters to remove harmonics. "Efficient" in the sense of good harmonic disturbance rejection and in the sense of low computational costs. The method is based on multirate filter banks (Strang and Nguyen, 1996). The concept of multirate filtering has first been introduced in (Kan and Aggarwal, 1972). A low order IIR filter can provide low deterioration at the frequency range of interest together with good robustness against small frequency variations of the harmonic frequencies. First, the structure and design of the filter bank will be presented. After that, some examples will be discussed. Finally, conclusions will be drawn.

2. METHOD

2.1 *Multirate digital harmonic notch filter bank*

A digital filter with sampling rate f_s consisting of n notches placed at the frequencies

$$f_k = \frac{k}{n+1} f_s, \quad k = 1, 2, ..., n \tag{1}$$

suppresses n harmonics with the first harmonic at $\frac{f_s}{n+1}$. The simplest construction of such a digital harmonic notch (DHN) filter consists of a zero at $e^{j2\pi \frac{f_k}{f_s}}$ and a pole at $ae^{j2\pi \frac{f_k}{f_s}}$ for $k = 1, 2, ..., n$ and $a < 1$ for harmonic attenuation. If the same filter coefficients are used at lower rates, the notch width becomes larger. To obtain an equal notch width for the ith-rate filter of an m-rate filter bank, it can be easily derived that the zeros and poles z_k, p_k, $k = 1, 2, ..., n$ should be

$$z_k = e^{j2\pi \frac{f_k}{f_{s,i}}} \tag{2}$$

$$p_k = a^{(n+1)(i-1)} z_k \tag{3}$$

with sampling rate of the ith-rate filter

$$f_{s,i} = \frac{f_s}{(n+1)^{i-1}} \tag{4}$$

If a signal is filtered with a DHN filter at a sampling rate of f_s ($i = 1$) and the output of this filter is filtered by a similar filter at a sampling rate of $\frac{f_s}{n+1}$ ($i = 2$), obtained by downsampling/decimation (Strang and Nguyen, 1996), then a 2-rate DHN filter bank is obtained which attenuates the harmonic frequencies

$$f_k = \frac{k}{(n+1)^2} f_s, \quad k = 1, 2, ..., (n+1)^2 - 1 \tag{5}$$

This recursive process can be repeated resulting in an m-rate filter bank which filters

$$N = (n+1)^m - 1 \tag{6}$$

harmonics (see also Table 1) with frequencies

$$f_k = \frac{k}{(n+1)^m} f_s, \quad k = 1, 2, ..., N \tag{7}$$

N	m=1	m=2	m=3	m=4
n=1	1	3	7	15
n=2	2	8	26	80
n=3	3	15	63	255
n=4	4	24	124	624

Table 1. Total amount of harmonic frequencies N filtered by the m-rate filter bank containing n notches at each rate.

To clarify this recursive process, an example is shown in Fig. 1. In this example, attenuation of the first 15 harmonics is obtained by applying two 3rd order filters at a sampling rate of f_s and $\frac{1}{4} f_s$ respectively. It is obvious that this significantly reduces the computational costs compared to a 15th order filter at a sampling rate of f_s.

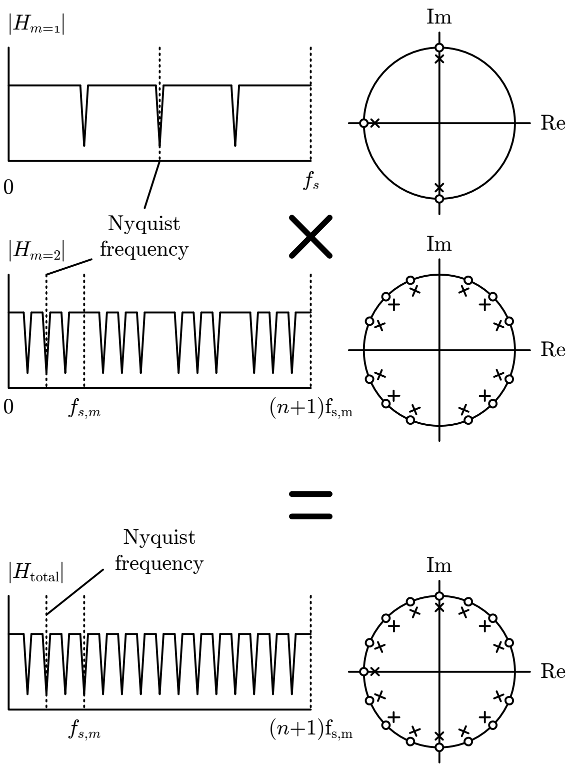

Fig. 1. Example of a multirate harmonic notch filter with $n = 3$ and $m = 2$. On the left the magnitude of $H_{m=1}$, $H_{m=2}$ and $H_{\text{total}} = H_{m=1} H_{m=2}$ is shown. The equivalent single rate filter with sampling rate f_s is shown on the right in terms of zeros "o" and poles "x" in the complex plane.

By choosing values for f_s, n and m, we obtain