ROBUST H_∞-CONTROL FOR MULTI-MASS SYSTEMS BASED ON A REDUCED MODEL

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Abstract: Elastically coupled multi-mass systems can be found in many mechanical applications, e.g. in robotics or machine tools. Parameter uncertainties often exist and desire a certain robustness of the control. An H_∞ control is an effective tool to control such systems. Unfortunately it is usually of a high order. While order reduction algorithms can be applied, the robustness of the resulting controller can no longer be guaranteed and has to be checked separately. In this paper an H_∞ control based on a reduced substitute model is presented. This way the order is kept low, while the robustness can still be guaranteed.

Topic: B - Control of mechatronic systems

Keywords: Robust control, Reduced-order models, Order reduction

1. INTRODUCTION

An important practical control application is the position control of a load mass coupled to a gear and the drive via elastic shafts. Such systems are used for example, in robotics and machine tools. The principle structure of a multi mass system is shown in the upper half of Fig 1.

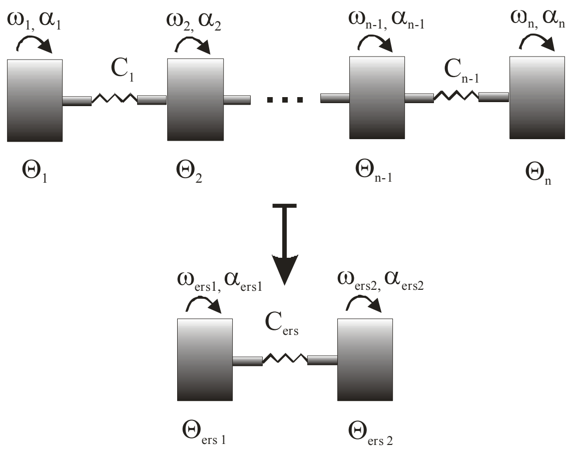

Fig 1: Model reduction

The masses coupled by elastic shafts form a system capable of torsion vibrations. Additionally the system may have backlash and friction, which make the system nonlinear. Furthermore not all the parameters are exactly known. Therefore the mathematical description of the multi-mass system will be subject to parameter uncertainties and will neglect some dynamics. As a result it is suggested to use a robust controller. An H_∞-control is a powerful tool to control linear systems with parameter uncertainties. With the help of a mixed sensitivity approach robustness and performance can be demanded separately of each other. Once the control is designed no further tuning is needed and stability can be guaranteed. The H_∞-controller is however usually of relatively high order. To reduce the controller order some order reduction algorithm can be applied to the controller (Balas *et al*., 2005). This has the disadvantage that the robustness can no longer be guaranteed and has to be checked separately.

In this paper a method is presented in order to reduce the order of the controller. The main idea is to create a controller on the basis of a two-mass system (ref. Fig 1). The neglected dynamics of the multi-mass system will be interpreted as parameter uncertainties for the controller synthesis. This way a robust H_∞-controller of low order can be created and the stability for the multi-mass system can be guaranteed. The design of the controller is shown in this paper for an exemplary three-mass system.

2. SYSTEM MODEL

2.1. Analytical description of the system

The block diagram of a multi-mass system is shown in Fig 2. The dynamics of the frequency converter and the electric part of the drive are approximated as a PT_1-element with the time constant T_a. The friction torque m_{R1} and the shaft torque of the first shaft m_1 are subtracted from the actual drive torque m_a. The resulting torque acts on the first mass with the moment of inertia Θ_1, resulting in the angular velocity ω_1 and after another integration in the position angle of the first mass α_1. Subtracting from α_1 the position angle of the second mass α_2 leads to the difference angle $\Delta\alpha_{12}$, which is input for the backlash function with the backlash angle α_{L1}. The resulting torsion angle is multiplied with the spring constant C_1 of the first elastic shaft resulting in the shaft torque m_1. This torque acts on the second mass with the moment of inertia Θ_1, leading to the angular velocity of the second mass ω_2, and so forth. At the last mass, a disturbance in the form of a load torque m_L may also appear.

Instead of taking the difference of the position angles $\Delta\alpha_{n,n+1} = \alpha_n - \alpha_{n+1}$ the angular velocities ω_n and ω_{n+1} can be subtracted from each other and this difference is then integrated. Thereby the general structure of a multi mass system can be described as composed of blocks for each mass as seen in Fig 2.

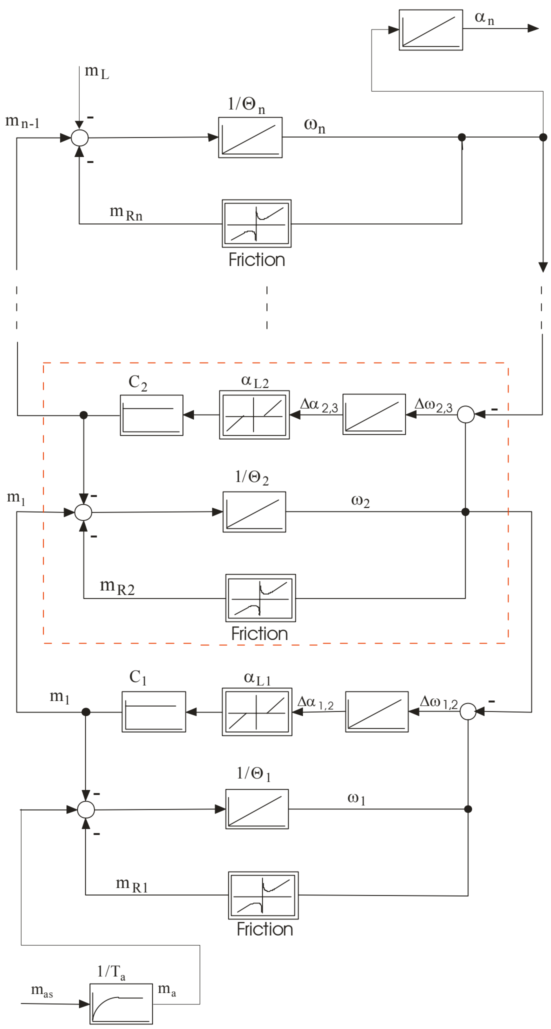

Fig 2: Block diagram of a multi mass system

2.2. Linear state space description

For the analytic state space description of the multi-mass system, the backlash and coulomb friction are neglected.

$$\dot{\vec{x}} = \underline{A}\vec{x} + \underline{B}u; \quad \vec{y} = \underline{C}\vec{x} + \underline{D}u \tag{1}$$

with

$$\underline{A} = \begin{pmatrix} \frac{-R_{L1}}{T_1} & \frac{-C_{N1}}{T_1} & 0 & \frac{C_{N1}}{T_1} & 0 & 0 & \cdots & 0 \\ \omega_0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \frac{C_{Ni-1}}{T_i} & \frac{-R_{Li}}{T_i} & \frac{-\left(C_{Ni-1}+C_{Ni}\right)}{T_i} & 0 & \frac{C_{Ni}}{T_i} & \cdots & 0 \\ \cdots & 0 & \omega_0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & & \cdots & & 0 & \frac{C_{Nn-1}}{T_n} & \frac{-R_{Ln}}{T_n} & \frac{-C_{Nn-1}}{T_n} \\ 0 & & \cdots & & 0 & 0 & \omega_0 & 0 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} \omega_{N1} \\ \alpha_{N1} \\ \vdots \\ \omega_{Ni} \\ \alpha_{Ni} \\ \vdots \\ \omega_{Nn} \\ \alpha_{Nn} \end{pmatrix}; \quad y = \alpha_{Nn}; \quad \underline{B} = \begin{pmatrix} \frac{1}{T_1} \\ 0 \\ \vdots \\ 0 \end{pmatrix}; \quad \underline{C} = \begin{pmatrix} 0 & \cdots & 0 & 1 \end{pmatrix}; \quad u = m_a$$

Here ω_0 is the nominal speed of the drive, R_{Li} are the linear friction constants with respect to speed and all parameters are normalized.

2.3. Substitute-model

To synthesize a controller of low order a substitute-two-mass model is derived from the analytical description of the multi-mass system (Fig 1).

This two-mass model is described by the following matrices.

$$\underbrace{\begin{pmatrix} \dot{\omega}_1 \\ \dot{\alpha}_1 \\ \dot{\omega}_n \\ \dot{\alpha}_n \end{pmatrix}}_{\dot{\vec{x}}_{ers}} = \underbrace{\begin{pmatrix} \frac{-R_{Lers1}}{T_{ers1}} & \frac{-C_{Ners}}{T_{ers1}} & 0 & \frac{C_{Ners}}{T_{ers1}} \\ \omega_0 & 0 & 0 & 0 \\ 0 & \frac{C_{Ners}}{T_{ers2}} & \frac{-R_{Lers2}}{T_{ers2}} & \frac{-C_{Ners}}{T_{ers2}} \\ 0 & 0 & \omega_0 & 0 \end{pmatrix}}_{\underline{A}_{ers}} \cdot \underbrace{\begin{pmatrix} \omega_1 \\ \alpha_1 \\ \omega_n \\ \alpha_n \end{pmatrix}}_{\vec{x}_{ers}} + \underbrace{\begin{pmatrix} \frac{1}{T_{ers1}} \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{\underline{B}_{ers}} \cdot \underbrace{m_a}_{u} \tag{2}$$