MODELLING DISTRIBUTED MANUFACTURING SYSTEMS VIA FIRST ORDER HYBRID PETRI NETS

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Abstract: A Distributed Manufacturing System (DMS) is a collection of independent companies possessing complementary skills and integrated with transportation and storage systems. This paper proposes a new model for DMS employing first order hybrid Petri nets, i.e., Petri nets based on first order fluid approximation. More precisely, transporters and manufacturers are described by continuous transitions, buffers are continuous places and products are represented by continuous flows routing from manufacturers, buffers and transporters. Moreover, discrete events occurring stochastically in the system are considered to take into account the start of the retailer requests and the blocking of transports and raw material supply. With the aim of showing the model effectiveness, a DMS example is modelled and simulated under two different operative conditions.

Keywords: manufacturing systems, Petri-nets, dynamic models, simulation, performance indices.

1. INTRODUCTION

A Distributed Manufacturing System (DMS) is a collection of independent companies possessing complementary skills and integrated with transportation and storage systems (Viswanadham and Raghavan 2000). Appropriate modelling and analysis of these highly complex systems are crucial for performance evaluation and comparison of competing DMS. However, few contributions face the problem of modelling the DMS in order to analyze the system performance measures and to optimize its functional objectives. Viswanadham and Raghavan (2000) model the system as a Discrete Event Dynamical System (DEDS), in which the evolution depends on the interaction of discrete events. Generalized Stochastic Petri Nets (GSPN) model a particular example of SC and determine the decoupling point location, i.e., the facility from which all finished goods are assembled after customer order confirmation. Moreover, in (Desrochers *et al.* 2005) a two product SC is modelled by complex-valued token Petri nets and the performance measures are determined by simulation. In addition, Dotoli and Fanti (2005) propose a GSPN model in order to describe in a modular way a generic DMS. However, the limit of this formalism is the modelling of products by means of discrete quantities (i.e., tokens). This assumption is not realistic in large systems with a huge amount of material flow. Since DMS are DEDS whose number of reachable states is very large, approximating fluid models can be used in this context as in manufacturing systems.

The aim of the paper is to propose a new model for DMS employing First Order Hybrid Petri Nets (FOHPN, Balduzzi *et al.* 2000), that include continuous places holding fluid and discrete places containing a non-negative integer number of tokens and transitions, where the latter are either discrete or continuous. Such a hybrid Petri net model is based on the framework proposed by Alla and David (1998) and presents the main key feature of allowing the Instantaneous Firing Speeds (IFS) of the continuous transitions to be chosen in a given range by a control agent. Moreover, the set of all admissible IFS vectors is explicitly characterized by the feasible solutions of a linear constraint set. Furthermore, an optimal IFS vector can be chosen according to a given objective function. Using such a modelling approach, this paper develops an FOHPN model of DMS by means of first-order fluid approximations. In particular, transporters and manufacturers are described by continuous transitions, buffers are continuous places and products are represented by continuous flows (fluids) routing from manufacturers, buffers and transporters. The model is built by using a modular approach based on the idea of the bottom-up methodology (Zhou and Venkatesh, 1998). A representative example, including the typical DMS elements, shows the effectiveness of the modelling technique that allows us to evaluate the system performance indices by the simulation.

The paper is structured as follows. Section 2 describes the structure and the dynamics of a generic DMS. Section 3 reports a brief overview of the FOHPN modelling formalism and Section 4 presents the modular DMS model. Section 5 describes and analyzes an example of DMS and a conclusion section closes the paper.

2. THE SYSTEM DESCRIPTION

A DMS may be described as a set of facilities with materials that flow from the sources of raw materials to subassembly producers and onwards to manufacturers and consumers of finished products. The DMS facilities are connected by transporters of materials, semi-finished goods and finished products. More precisely, the entities of a DMS can be summarized as follows.

1- *Suppliers:* a supplier is a facility that provides raw materials, components and semi-finished products to manufacturers that make use of them.
2- *Manufacturers and assemblers*: manufacturers and assemblers are facilities that transform input raw materials/components into desired output products.
3- *Logistics and transporters*: storage systems and transporters play a critical role in distributed manufacturing. The attributes of logistics facilities are storage and handling capacities, transportation times, operation and inventory costs.
4- *Retailers or customers*: retailers or customers are sink nodes of material flows.

Here, part of the logistics, such as storage buffers, is considered pertaining to manufacturers, suppliers and customers. Moreover, transporters connect the different stages of the production process.

The dynamics of the distributed production system is traced by the flow of products between facilities and transporters. Because of the large amount of material flowing in the system, we model a DMS as a hybrid system: the continuous dynamics models the flow of products in the DMS, the manufacturing and the assembling of different products and its storage in appropriate buffers. Hence, resources with limited capacities are represented by continuous states describing the amount of fluid material that the resource stores.

Moreover, we consider also discrete events occurring stochastically in the system, such as:

a) the blocking of the raw material supply, e.g. modelling the occurrence of labour strikes, accidents or stops due to the shifts;
b) the blocking of the transport operations due to the shifts or to unpredictable events such as jamming of transportation routes, accidents, strikes of transporters etc.;
c) the start of a request from the retailers.

3. FIRST-ORDER HYBRID PETRI NETS

3.1 The net structure and marking.

This section recalls the First Order Hybrid Petri Nets (FOHPN) formalism used in the following (Balduzzi *et al.* 2000).

A FOHPN is a bipartite digraph described by the six-tuple $PN=(P, T, Pre, Post, D, F)$. The set of places $P=P_d \cup P_c$ is partitioned into a set of discrete places P_d (represented by circles) and a set of continuous places (represented by double circles).

The set of transitions $T=T_d \cup T_c$ is partitioned into a set of discrete transitions T_d and a set of continuous transitions T_c (represented by double boxes). Moreover, the set of discrete transitions $T_d=T_I \cup T_E$ is further partitioned into a set of immediate transitions T_I (represented by bars) and a set of exponentially distributed transitions T_E (represented by boxes).

The matrices *Pre* and *Post* are the pre-incidence and the post-incidence matrices, respectively, of dimension $|P|\times|T|$. Note that the symbol $|A|$ denotes the cardinality of set A. Such matrices specify the net digraph arcs and are defined as follows:

$$Pre, Post : \begin{cases} P_c \times T \to \mathbb{R}^+ \\ P_d \times T \to \mathbb{N} \end{cases} .$$

We require that for all $t \in T_c$ and for all $p \in P_d$ it holds $Pre(p,t)=Post(p,t)$ (*well-formed nets*).

The function D: $T_t \to R^+$ specifies the timing associated to exponentially distributed timed transition $t_j \in T_E$. More precisely, we associate to each $t_j \in T_E$ the average firing delay $\delta_j = D(t_j)$. Moreover, the function $F : T_c \to R^+ \times R_\infty^+$ specifies the firing speeds associated to continuous transitions (we denote $R_\infty^+ = R^+ \cup \{\infty\}$). For any continuous transition $t_j \in T_c$ we let $F(t_j)=(V_{mj}, V_{Mj})$, with $V_{mj} \le V_{Mj}$. Here, V_{mj} represents the minimum firing speed (mfs) and V_{Mj} the Maximum Firing Speed (MFS) of the generic continuous transition.

Given a FOHPN and a transition $t \in T$, the following sets of places may be defined: $\bullet t=\{p \in P: Pre(p,t)>0\}$, named pre-set of t; $t\bullet=\{p \in P: Post(p,t)>0\}$, named post-set of t. Moreover, the corresponding restrictions to continuous or discrete places are defined as ${}^{(d)}t=\bullet t \cap P_d$ or ${}^{(c)}t=\bullet t \cap P_c$. Similar notations may be used for pre-sets and post-sets of places. The incidence matrix of the net is defined as $\mathbf{C}(p,t)=Post(p,t)-Pre(p,t)$. The restriction of $\mathbf{C}$ to P_X and T_X (with X,Y$\in$ {c, d}) is denoted by $\mathbf{C}_{XY}$.

A marking

$$\mathbf{m} : \begin{cases} P_d \to \mathbb{N} \\ P_c \to \mathbb{R}^+ \end{cases}$$