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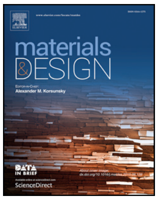

Prediction of forming limit curves for nonlinear loading paths using quadratic and non-quadratic yield criteria and variable imperfection factor

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ABSTRACT

Industrial sheet metal forming processes often involve complex deformation modes and it is necessary to consider nonlinear loading path effects when predicting forming limit curves. Moreover, the yield criterion plays a critical role in the accuracy of predicted forming limits.

In this work the MK analysis was modified to relate the initial imperfection factor to a physical property such as the surface roughness, and the orientation of the imperfection was also allowed to vary. This model was used to predict the strain-based and stress-based forming limit curves (FLC and SFLC) of sheet materials that are subject to either linear or non-linear strain paths.

Two different yield criteria were employed in this study, Hill's 1948 quadratic yield criterion and Hosford's 1979 non-quadratic yield criterion. The theoretical model was validated by comparing predicted FLC and experimental FLC curves obtained from the literature. FLCs and SFLCs predicted with these two yield criteria were compared for both linear and nonlinear loading paths.

Results showed that both the quadratic and non-quadratic yield criteria predict the FLC with acceptable accuracy however on the whole the non-quadratic yield criterion generally provides a slightly better correlation with experimental data, especially on the right side of the FLC.

1. Introduction

Significant progress in sheet metal formability evaluation occurred in Ref. [22] reported that localized necking in stretched sheets requires a critical combination of major and minor in-plane strains (along two perpendicular directions in the plane of the sheet). Subsequently, this concept was extended by Goodwin [11] to drawn sheet and the resulting curve is known as the Keeler–Goodwin curve, or more commonly, the forming limit curve (FLC). In other words, combinations of principal strains that lie above the FLC present some risk of necking, while those that lie below lead to a safe process. The FLC has become an essential tool to evaluate sheet formability, and it is typically obtained by stretching gridded sheet specimens of various widths over a hemispherical punch.

However, the deformation behavior of metals is strongly dependent on the history of loading, in particular, on the specific strain path. The FLC, as a well established aid to either experimental or theoretical studies of the formability of sheet metal, should therefore be represented in terms of specific strain history.

Although the FLC has been successfully used to evaluate sheet forming processes for many years, it has been shown that it is only valid for quasi-linear strain paths. Non-linear strain paths cause the FLC to translate in strain space, which can lead to erroneous interpretations of the forming severity for multi-stage processes in which the strain path is significantly non-linear and this has been investigated for all sheet materials including steel, copper and brass, as reported, for example, by Kleemola and Pelkkikangas [23].

It has already been shown by Stören and Rice [36] that in FLC prediction and generally when modeling the plastic behavior of metals, the yield function, which is usually assumed to take the same form as the plastic potential function in classical plasticity theory, plays an important role. It determines the direction of the plastic strain increment via the associated flow rule, and consequently the value of the effective plastic strain which in turn determines the work-hardening rate as defined by the work hardening function.

Parmar and Mellor [33] investigated the discrepancy between theoretical and experimental results for aluminum alloys in metal forming calculations and concluded that it was due to the inadequacy of Hill's [12] yield criterion to represent materials with anisotropy coefficients (r_0, r_{45}, and r_{90}) lower than unit. They recommended employing Hill's [14] non-quadratic yield criterion for the prediction of the FLC of aluminum and other alloys with lower r-values.

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Barlat [6] conducted a study on the effect of the shape of the yield surface on limit strains. In this investigation, Barlat listed critical characteristics of the yield surface and defined a new parameter, *P*, as the ratio of the yield stress in plane-strain to the yield stress in equibiaxial tension.

Lian et al. and Xu et al. [24,41] employed Hill's [14,15] yield criteria and the MK analysis to predict the right side of the FLC where both major and minor in-plane strains are positive. They compared predicted forming limit results with corresponding experimental data for both aluminum and AK steel. Results showed that by using these yield criteria limit strains can be reasonably predicted. Also Asaro and Needleman [1] and Tvergaard and Needleman [37] introduced an alternate method to study the effects of plastic anisotropy on localized necking. They used an elastic-viscoplastic Taylor-type polycrystalline approach for initial texture representation and accounted for the texture evolution during on-going plastic deformation. This method was applied later by Wu et al. [38,39] to predict localized necking in rolled aluminum alloys.

Hora et al. [16] made some improvements to Swift's diffuse necking criterion, and with the help of some experimental research, confirmed that the strain path, i.e. the ratio of the minor strain component (ε_2) to the major strain component (ε_1), is the most important factor to determine the onset of necking in sheet metals. Kuroda and Tvergaard [21] used different orthotropic yield criteria in FLC prediction and they concluded that orthotropic axes disorientation may have an effect on predicted limit strains. Cao et al. [9] predicted limit strains using the MK analysis and the Karafillis–Boyce anisotropic yield criterion for the right side of the FLC and offered a new approach to specify yield criterion constants. Banabic and Dannenmann [3] used Hill's 93 yield criterion to study the influence of parameter *a* (defined as the ratio of the uniaxial yield stress to the biaxial yield stress) on limit strains using MK analysis and Swift's bifurcation instability theory. Using both methods they showed that the FLC translates upward, especially in equibiaxial tension, when parameter *a* is increased. Butuc et al. [7] examined the performance of two non-quadratic yield functions, Yld96 and BBC2000 in FLC prediction. The correlation of theoretical results and experimental data was shown to be satisfactory when using these yield criteria. Banabic et al. [2] compared the accuracy of a variety of FLC prediction methods using the orthotropic yield criterion BBC2003: in their work FLCs were predicted using Swift's diffuse necking criterion, Hill's bifurcation theory, the finite element method (FEM), the MK analysis and the method proposed by Hora et al. [16]. Banabic et al. [4] showed that the MK analysis and the FEM method gave a better correlation with experimental data than the other methods.

In the current research both Hill's [12] quadratic yield function and Hosford's [17] non-quadratic yield function were employed in a modified MK analysis [26,27] to predict strain-based and stress-based forming limit curves following both linear and nonlinear loading paths. FLCs were calculated for AISI-1012 steel and AA-2008-T4 aluminum sheets and were compared with published experimental data [10,28].

2. Theory

2.1. MK analysis for prediction of FLC

One of the most effective methods to predict the onset of localized deformation was introduced by Marciniak and Kuczynski [26,27] and is now commonly known as the MK method. This approach is based on the assumption that the inherent material heterogeneities in a thin sheet can be modeled by a very shallow groove. After a certain amount of deformation the strain in the groove increases more rapidly than elsewhere and a localized neck inevitably develops from this initial imperfection (Fig. 1). Due to its simplicity, the MK method has been used with different plasticity theories and hardening models to predict history-dependent forming limits [8,42].

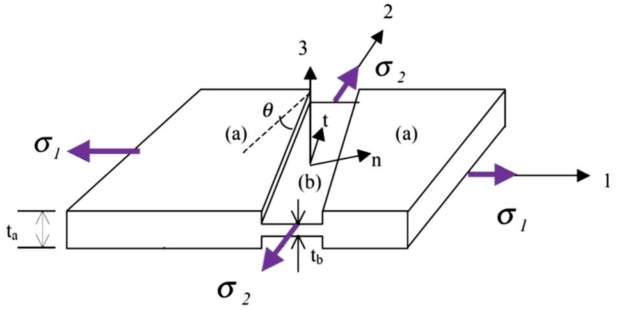

Fig. 1. Thickness imperfection in the MK method.

McCarron et al. [29] presented the fundamentals of the MK analysis with a significant level of detail. These researchers machined grooves of different depths into samples made from two different grades of steel as a representation of the imperfection region in the MK method. These samples were then subjected to balanced biaxial tension, and the results of their investigation showed that there is no reduction of the limiting strains for very shallow grooves. In other words, when the ratio of the thickness in the groove to that of the sheet is greater than 0.992, the forming limit strains remain unchanged. This imperfection factor is considered equivalent to the microstructural defects that normally exist in as-rolled metal sheets.

In the MK model, a sheet with a nominal thickness is assumed to have a band (in the shape of a groove) that is slightly thinner than the rest of the sheet; these two areas are denoted by (a) and (b), respectively (Fig. 1). In the current work, the initial imperfection factor of the groove, f_o, was defined as the thickness ratio as follows:

$$f_o = \frac{t_o^b}{t_o^a} \tag{1}$$

where '*t*' denotes the sheet thickness, and subscript '*o*' denotes the initial state. As deformation progresses the updated thickness imperfection can be determined from Eq. (1):

$$\frac{\mathrm{d}f}{f} = \mathrm{d}\varepsilon_3^b - \mathrm{d}\varepsilon_3^a \tag{2a}$$

$$f = f_o \exp\left(\varepsilon_3^b - \varepsilon_3^a\right) \tag{2b}$$

where 'ε_3'denotes the true thickness strain. In this work, the imperfection factor was considered to change with the deformation of the sheet. In order to estimate the initial imperfection factor, it was thought reasonable to relate it to the surface roughness of the sheet. By assuming that the maximum thickness difference between regions (b) and (a) is equal to the surface roughness of the sheet, the initial imperfection factor can be written as follows:

$$f_o = \frac{t_o^a - 2R_{zm}}{t_o^a} \tag{3}$$

where R_{zm} is the maximum surface roughness of the sheet.

Research carried out by Stachowicz [35] shows that the surface roughness also changes with deformation and these changes depend upon the initial surface roughness, the grain size, and the strain according to the following empirical relation:

$$R_{zm} = R_{zo} + C d_o^{0.5} \varepsilon_e^b \tag{4}$$

where 'R_{zo}'is the surface roughness before deformation, C is a material constant, ε_e is the effective strain, and d_o is the initial grain size. Combining Eqs. (2a), (2b), (3) and (4) yields:

$$f_o = \frac{t_o^a - 2\left[R_{zo} + C d_o^{0.5} \varepsilon_e^b\right]}{t_o^a} \tag{5a}$$