Materials and Design 87 (2015) 278–286

Contents lists available at ScienceDirect

Materials and Design

journal homepage: www.elsevier.com/locate/jmad

Collision and rebound of ping pong balls on a rigid target

R.H. Bao [a,*], T.X. Yu [a,b]

[a] Department of Engineering Mechanics, Zhejiang University, Hangzhou, Zhejiang 310027, PR China
[b] Department of Mechanical and Aerospace Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

A R T I C L E   I N F O

Article history:
Received 3 May 2015
Received in revised form 31 July 2015
Accepted 4 August 2015
Available online 8 August 2015

Keywords:
Hollow ball
Buckle
Rebound
Coefficient of restitution

A B S T R A C T

The collision and rebound behavior of ping pong balls impinging onto rigid target are studied. Three dimensionless dominant parameters are identified: (1) the ratio of the wall-thickness to the average radius of the ball; (2) the dimensionless initial velocity; and (3) the yield strain of the material.

Depending on the dimensionless initial velocity, various collision and rebound behaviors of the ball are revealed: (1) When the initial velocity is low, the deformation of the ball remains purely elastic, for which the characteristic duration is theoretically obtained; (2) With the increase of initial velocity, the ball's cap begins to buckle and multiple impacts occur, leading to the increase of the restitution duration and the reduction of the coefficient of restitution (COR); (3) With the higher initial velocity, the ball's cap buckles permanently, leading to the disappearance of multiple impacts and a sudden drop of the restitution duration; consequently the COR decreases from 0.5 to 0.3; and (4) When the initial velocity is close to the material's yield velocity, the ball buckles into a non-axisymmetric mode.

The simulation results are also compared with experimental ones. Furthermore, the effects of thickness-to-radius ratio, yield strain and coefficient of friction are also discussed.

1. Introduction

The impact of a freely flying body or a freely flying structural component impinging onto a stationary solid target widely occurs in daily life and engineering, ranging from macroscopic scales [1–5] to microscopic scales [6,7]. As pointed out by Stronge [4], the initial kinetic energy carried by the flying body is transformed into the internal energy of the deformable body during the compression phase, and then the stored elastic strain energy is progressively recovered into kinetic energy during the subsequent restitution phase through the contact force. Coefficient of restitution (COR) plays a key role in measuring the global energy loss during the collision, as results of stress wave propagation, material's viscosity and possible plastic deformation, etc. COR is usually defined by one of the following three ways:

$$e_1 = \frac{V_r}{V_i} \tag{1a}$$

$$e_2 = \frac{I_R}{I_C} = \frac{\int_{t_R} Fdt}{\int_{t_C} Fdt} \tag{1b}$$

$$e_3 = \sqrt{\frac{E_r}{E_i}} \tag{1c}$$

where e_1, e_2, and e_3 denote the Newtonian (kinematic), Poisson (kinetic) and energetic COR, respectively, V_r and V_i are the relative velocities of the colliding bodies after and before the collision, I_C and I_R are the impulses over the compression duration t_C and the restitution duration t_R, E_r and E_i are the kinetic energies of the colliding bodies after and before the collision, respectively. Among these expressions, definition (1a) is the most straightforward and widely applied one, and therefore is employed in this paper.

The collisions of a solid sphere to another solid sphere and those between a solid sphere and a substrate have attracted extensive attention over the last few decades. Hunter [8] theoretically analyzed the elastic impact of a solid ball on a stationary infinite half space, and found that less than 3% of the initial kinetic energy is lost in the form of elastic stress wave. Reed [9] further improved the Hunter's solution and concluded that the energy loss during elastic impact is greater than that predicted by Hunter. Tillett [10] experimentally investigated the energy loss due to stress wave propagation, and found that it was in the order of 3% for a steel ball impinging on glass substrate. Hutchings [11] developed a model to include the plastic deformation and showed that the energy loss of elastic stress waves is about 3% for hard steel spheres impinging on a mild steel target with velocities of about 70 m/s. Wu et al. [12,13] systematically analyzed the energy loss of elastoplastic solid spheres impinging on a half-space with finite element method. All

---

[*] Corresponding author.
E-mail address: brh@zju.edu.cn (R.H. Bao).

http://dx.doi.org/10.1016/j.matdes.2015.08.019

these studies show that energy loss in the form of elastic stress wave during collisions of solid bodies is negligible.

However, for the collision of thin-walled structures, elastic bending wave and/or structural vibration would play significant roles during collision and rebounding. Bao and Yu [14] systematically investigated the collision of a freely flying elastic–plastic circular ring on a rigid target by employing finite element method. They found that even if the ring only experiences elastic deformation during the collision, about 7/16 of the initial kinetic energy of the ring would transform into the energies carried out by elastic bending wave and elastic vibration after the ring rebounds from the rigid target, and this results in the COR being about 3/4. Xu et al. [15] then conducted an experimental study to record the collision and rebounding process of freely flying aluminum rings impinging on a hard anvil with initial velocity ranged from 15 m/s to 115 m/s. Both the numerical results by Bao and Yu [14] and experimental results by Xu et al. [15] showed that during the free collision the ring underwent a global deformation due to the effect of stress wave, making the deformation modes more diversified than that under static compression, while the resulted COR is far below that resulted from solid body collisions.

Thin-walled spheres (hollow balls) or sphere arrays also have wide applications as energy absorbers [16,17] or in the sports games [18, 19]. When a hollow sphere is subjected to various types of loading, its deformation is much more complicated than that of a solid ball, as revealed by numerous studies in the last decades. Both the static compression and dynamic crushing of hollow balls have been extensively studied by theoretical [20–23], numerical [24] and experimental [24, 25] means. However, little work has been devoted to the collision of freely flying hollow balls up to now. It is noticed that Cross systematically studied the dynamic and rebound behaviors of a variety of sports/game balls, including tennis ball, squash balls, golf ball, baseball, ping pong ball and so on [18,19,26,27]. Hubbard and Stronge [28] carried out an experiment of a ping pong ball impinging onto a flat glass plate with the initial velocities lower than 20 m/s, and obtained the colliding duration and COR; however, their theoretical model was purely elastic and only flattening deformation mode was included, so the results were only applicable to soft balls. A research group led by T.X. Yu at HKUST studied the dynamic and crushing behavior of ping pong balls or ping pong arrays instead of metallic hollow balls [29–32]. Bao and Yu [33] numerically studied the crushing and rebounding behavior of elastoplastic hollow balls made of aluminum, which impinged a rigid target. They found that the COR of a hollow ball after a collision was significantly lower than that of a solid ball, and the transition from axisymmetric dimpling to non-axisymmetric lobes depended on the initial velocity of the ball as well as the geometrical and material properties. In fact, Cross [19] measured the colliding duration and impact force of ping pong ball and squash ball with piezoelectric ceramic plates, and found the significant difference between these two kinds of balls. Besides, Zhang et al. [34] developed a viscoelastic impact model to predict the dynamic behavior of a ping pong ball impinging on a rigid target and compared with the experimental results.

In the present paper, we will employ finite element method to systematically investigate the free impact of a flying ping pong ball on a rigid target, and reveal detailed dynamic deformation features of the ball, and in particular, the dependence of the COR on the initial velocities.

2. FEM models

Fig. 1a and b shows the schematic drawing and the finite element model of a ping pong ball impinging freely onto a stationary rigid plate with initial velocity, V_0, where R and h denote the average radius and wall-thickness of the ping pong ball, respectively.

Before the Sydney Olympic Games hold in 2000, the outer diameter and mass of an international standard ping pong ball were $D = 38$ mm and $m = 2.5$ g, respectively, and then were officially regulated by ITTF as $D = 40$ mm and $m = 2.7$ g to slow down the speed of ping pong balls during the play, making the game more attractive to audience [35]. Ping pong balls are usually made of celluloid, a lightweight and flexible plastic with high tensile strength. As reported in literature, the density of celluloid material is about $\rho = 1400–1600$ kg/m^3, therefore, the average wall-thickness for either 38 mm or 40 mm ping pong balls is about $h = 0.4$ mm, and the average radius is about $R = 18.6$ mm for 38 mm ball and $R = 19.6$ mm for 40 mm ball.

Owing to the inconsistent ingredients and manufacturing process, the reported mechanical properties of celluloid material vary from case to case and have not been provided officially. The elastic modulus adopted by Hubbard and Stronge [28] was 1.6 GPa for Spalding 3 Star 38 mm ping pong ball. Ruan et al. [31] carried out a tensile test on the dog-bone samples cut from 40 mm ping pong balls, and concluded that celluloid material exhibits very good linear elastic and perfect plastic behavior with the elastic modulus and yield stress being 2.4 GPa and 47 MPa, respectively. Robert et al. [36] measured the elastic modulus and yield stress by tensile test and dynamic mechanical thermal analyzer (DMTA) for 40 mm ping pong ball, which were about 2.2 GPa and 60 MPa, respectively. For simplicity, the celluloid of ping pong ball is assumed to be linear elastic and perfectly plastic in the present study, while the density, elastic modulus and yield stress of the material

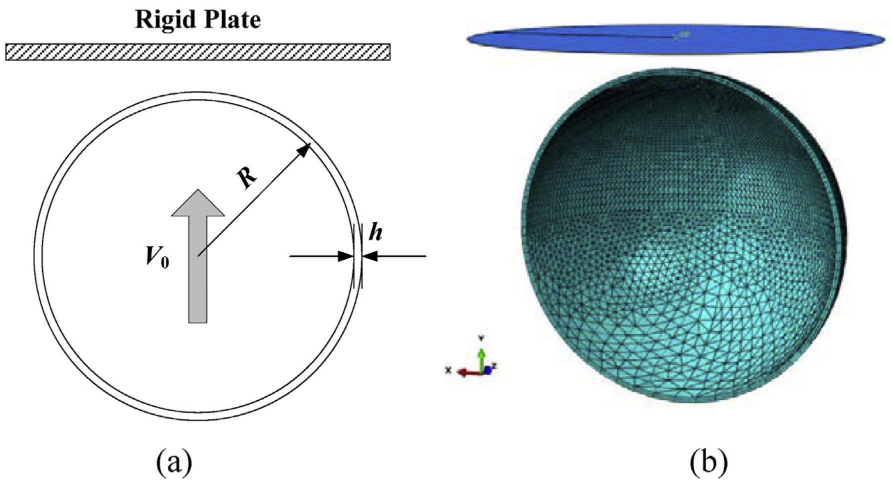

Fig. 1. Sketch (a) and FEM model (b) of a freely flying ping pong ball impinging onto a stationary rigid plate.