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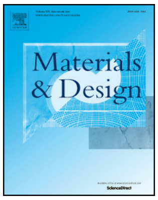

Evaluation of fracture in mortar subject to tension loading using phase field model and three point bending test

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ABSTRACT

The Classic Fracture Mechanics (CFM) is the most widely used approach to study the initiation and propagation of cracks, which has the limitations in numerical simulations due to complex topological changes and possible singularity. To overcome its limitations, the Phase-Field Model (PFM) is presented to model the cracking failure in mortar subject to pure tension loading (Mode I) with different water/cement ratio and thickness. The mortar cracking surfaces are described using a phase-field variable which assumes one in the intact region and negative one in the crack region. The new white noise term is added into the classical Phase-Field Model to reflect the quasi-brittle cracking behavior of mortar. The non-conserved Allen–Cahn dynamics is then employed to simulate the growth of cracks. To verify the simulation results, three point bending test is conducted. It is discovered that the crack initiation and propagation in our simulation agrees very well with the experimental results.

1. Introduction

Mortar is an important component in concrete. Its fracture properties have been investigated by both experimental studies, e.g., fracture properties of mortar [6,35], crack propagation in cementitious mortar [11], microcracking in cement mortar [25,26], crack deflection [31], and numerical analysis, such as Finite Element Modeling [20], non-extensive statistical modeling [27], and cohesive work approach [32]. Most of the current mortar cracking analyses are based on Classical Fracture Mechanics (CFM) originally proposed by Griffith [13]. Although CFM has shown remarkable progress in cracking analysis, it still has some limitations, e.g. possible multiple cracking criterion problem [19], complex topological transitions [10], singularity problem [8] etc.

Recently, a new energy-based model, Phase-Field Model (PFM), has been introduced to material analysis. This method is proposed by Cahn and Hilliard to study the phase transition [7] and phase transformation [29] and has been applied in a variety of problem analysis, such as hardening alloy [34], magnetoelectric composites [22], dendritic growth [36], and fracture analysis [12,18]. Since PFM could handle the cracking in a natural way as the competition between the growth of surface free energy and release of elastic energy in the direction of the steepest energy minimization descent, it has the potential to overcome limitations of CFM. There have been several versions of phase-field formulations for crack propagation, in which the phase field variable is used to identify the unbroken state and the fully broken state inside the crack. Both the non-conserved Allen–Cahn [15,18] and conserved Cahn–Hilliard dynamics [12] could solve the cracking problems satisfactorily. Christian and Lisa-Marie [9] studied the rubbery polymer brittle cracking using Phase-Field Modeling. Abdollahi and Arias [1] investigated the ferroelectric materials fracture based on a coupled Phase-Field Modeling. However, limited research has been conducted to study the fracture properties of cement mortar using PFM.

In this paper, PFM is used to analyze the cracking failure in mortar subject to pure tension loading (Mode I) with different water/cement ratio (w/c) and specimen thickness. Different from CFM, PFM describes the entire mortar cracking evolution only by total energy minimization principle and, thus, requires no further criterion. To simulate the mortar cracking process, the basic phase-field equations are first presented based on the Ginzburg–Landau theory [33]. Phase-field is then coupled with the elastic field in mortar cracking. To reflect the quasi-brittle cracking behavior, the classic Allen–Cahn equation [3] is modified by adding a new white noise term to evolve the phase-field variable. The fracture resistance is represented by surface free energy stored between crack phase and intact phase. The weak forms of the revised Allen–Cahn equations are then established. The mortar structure is described using a phase-field variable which assumes one in the intact solid and negative one in the crack region. A two dimensional plane strain simplification is used for approximation. By solving the governing equation using COMSOL Multiphysics, a commercial finite element software, the cracking process in mortar could be obtained. The numerical simulation is

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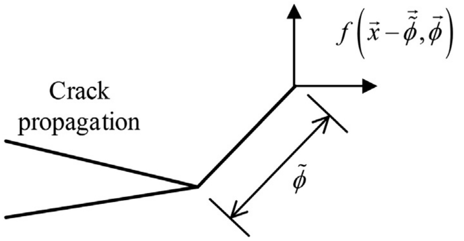

Fig. 1. Crack propagation in lab coordinates.

then validated by comparing with the Griffith cracking criterion and the three point bending test results.

2. Theoretical analysis

2.1. Phase-Field Model

In Phase-Field Modeling, the phase-field variable ϕ is used to describe the cracking surface, which is $\phi=-1$ for the broken phase, $\phi=+1$ for the intact phase, and a diffuse interface separates the two phases.

The Ginzburg–Landau form of total free energy in the system is obtained as [33]

$$F=\int_{\Omega}\left(f_{gr}+f_{dw}+W\right)dV \tag{1}$$

where the gradient energy density is $f_{gr}=\frac{1}{2}\lambda|\nabla\phi|^2$ [33], the local free energy density is a double-well potential $f_{dw}=\frac{\lambda}{4\epsilon^2}(1-\phi)^2(1+\phi)^2$ which has two minima at the crack void phase and the intact solid phase [33]. W is the elastic energy density.

The strain tensor ε_{ik} is obtained as,

$$\varepsilon_{ik}=\frac{1}{2}\left(u_{i,k}+u_{k,i}\right) \tag{2}$$

where u is the displacement field.

The stress must satisfy the force equilibrium equation

$$\nabla\cdot\boldsymbol{\sigma}=0. \tag{3}$$

Note that only the elastic energy density will directly contribute to the cracking process, which can be expressed as [16]

$$W=\frac{E(\phi)}{2(1+\nu(\phi))}\left[\frac{\nu(\phi)}{1-2\nu(\phi)}(\varepsilon_{ii}-\varepsilon_0)^2+(\varepsilon_{ik}-\varepsilon_0)^2\right] \tag{4}$$

where $E(\phi)$ is the elastic modulus and $\nu(\phi)$ is Poisson's ratio.

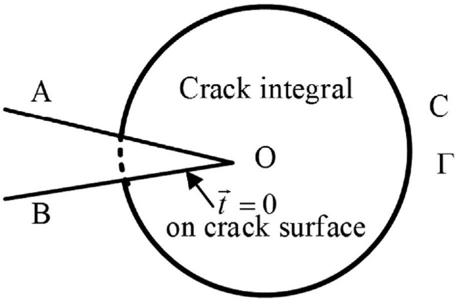

Fig. 2. Crack integral without energy flux on crack surface.

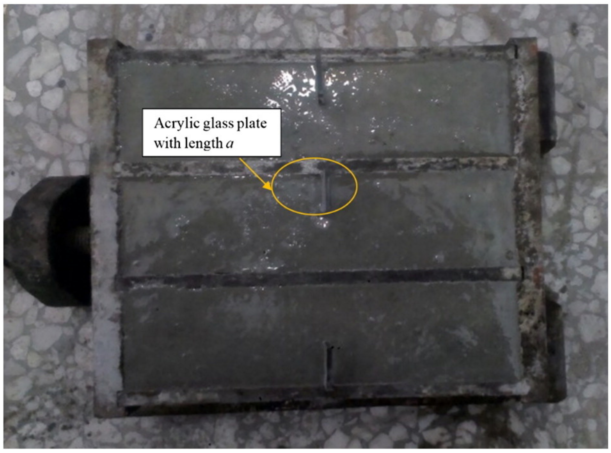

Fig. 3. Specimen preparation using iron mold and acrylic glass plate.

We express $E(\phi)$ as [30]

$$E(\phi)=E^{-1}+\left(E^{+1}-E^{-1}\right)h(\phi), \tag{5}$$

where E^{+1} and E^{-1} are the elastic moduli of the intact solid phase and the crack void phase respectively, and $h(\phi)$ is the fitting polynomial function

$$h(\phi)=\frac{3}{2}\phi^5-\frac{5}{2}\phi^3+2\phi \tag{6}$$

which satisfies $h(-1)=-1$, $h(1)=1$, $h'(-1)=h'(1)=0$ and interpolates the elastic modulus between E^{+1} and E^{-1}. Note that the rationale of these parameters has been explained in Hou et al. [16].

After we obtain the elastic energy, we can plug it into the Phase-Field Model. To describe the growth of cracks, the Allen–Cahn dynamics is employed to study the evolution of phase-field variable as [3]

$$\frac{\partial\phi}{\partial t}=-M\psi \tag{7}$$

where the mobility parameter M is used to controls the crack propagation speed, and ψ is the chemical potential calculated as

$$\psi=\frac{\delta F}{\delta\phi}=-\nabla\cdot\lambda\nabla\phi+\frac{\lambda}{\epsilon^2}\left(\phi^2-1\right)\phi+\frac{\partial W}{\partial\phi}. \tag{8}$$

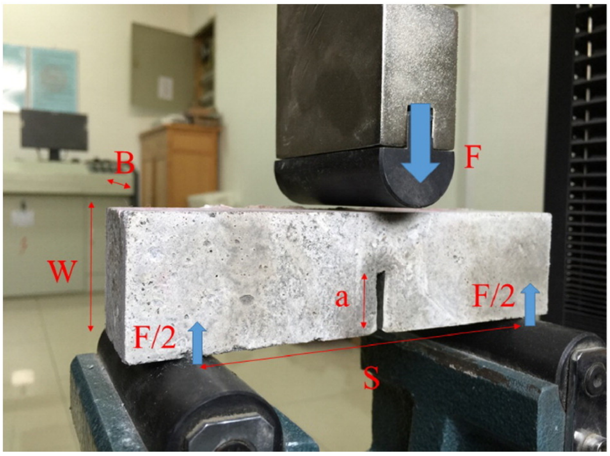

Fig. 4. Three point bending test on notched mortar specimen.