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A method for long-term creep–rupture strength prediction based on a small sample of experimental results, smoothed bootstrapping and time–temperature parameters

Domen Šeruga *, Marko Nagode

University of Ljubljana, Faculty of Mechanical Engineering, Aškerčeva 6, SI-1000 Ljubljana, Slovenia

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ABSTRACT

A method is presented that enables predictions of long-term creep–rupture strength based on a small sample of short-term experimental results. Smoothed bootstrapping in combination with the most commonly used time–temperature parameters is used for the determination of the optimal values of coefficients. For ten metals, creep–rupture strengths are evaluated for both full-size and sub-size data sets. Predictions in the case of small data sets always lie on the conservative side, whereas the confidence interval of predicted strengths decreases with an increasing number of experimental results. However it is shown here that for the evaluated materials, usable interim predictions of creep–rupture strengths can be achieved.

1. Introduction

Thermal and mechanical loadings can cause fatigue and creep failures of products operating under variable conditions (e.g. an automotive exhaust system [1–6]). Both fatigue and creep mechanisms can affect a product at any time during operation. However, whilst the fatigue mechanism is significant under any temperature regime, the creep mechanism is especially intense as the temperature rises [7–10]. Many creep–rupture models exist, and parametric models in particular play an important role in the prediction of rupture strengths [11]. Since their introduction, first by Larson and Miller [12], Manson and Haferd [13] and later by Orr et al. [14], time–temperature parameters have become a common method of describing creep–rupture data [11,15,16]. A variety of other time–temperature parameters have been developed since, i.e. Manson–Succop [17], Goldhoff-Sherby [18], Soviet model [19], Minimum commitment model [20], etc. each suggesting a different description of master curves representing the relation between logarithmic stress $\log \sigma$ and logarithmic time to rupture $\log t_r$ for temperature T. In addition to the rupture strength, some creep–rupture models also describe the change in creep strain up to the point of rupture, i.e. Theta projection model [21] and the Wilshire approach [22].

The Restrained–Manson–Brown (RMB) parameter [10,23],

$$\mathrm{RMB} = \frac{\log t_r - \log t_a \cdot T^{|q|-1}}{(T - T_a \cdot \langle q \rangle)^q}, \tag{1}$$

has been introduced as one of the approaches for unification of the most commonly used time–temperature parameters and has been proposed as a modified form of the well-known Manson–Brown (MB) parameter [24]. The RMB parameter therefore resembles all the properties of the MB parameter but is numerically advantageous to solve because the coefficients $\log t_a$, T_a and q can be gained by solving a set of linear equations rather than nonlinear equations, as is the case for the MB parameter. Depending on the value of the coefficient q, the RMB parameter can represent the Manson–Haferd (MH) parameter ($q = 1$),

$$\mathrm{MH} = \frac{\log t_r - \log t_a}{T - T_a}, \tag{2}$$

the Orr–Sherby–Dorn (OSD) parameter ($q = 0$),

$$\mathrm{OSD} = \log t_r - \frac{\log t_a}{T}, \tag{3}$$

or the Larson–Miller (LM) parameter ($q = -1$),

$$\mathrm{LM} = T(\log t_r - \log t_a). \tag{4}$$

In addition, the RMB parameter can also suggest its own solution (Eq. (1)) if $q \neq \{1, 0, -1\}$. The calculated values of coefficients always

* Corresponding author. Tel.: +386 1 4771 743; fax: +386 1 2518 567.
E-mail address: domen.seruga@fs.uni-lj.si (D. Šeruga).

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ensure the best fit to the input data. Yet the quality of every creep–rupture model depends on the range and the duration of the available test results and their scatter. Although mechanical components can be designed to operate for up to 250,000 h under variable conditions, creep–rupture tests are usually performed for much shorter durations and only for a finite number of test temperatures and stresses [24,25]. Furthermore, even under controlled testing, a considerable scatter of test results is observed [24,26]. Thus a good model will reliably predict the time to rupture or the rupture strength inside and outside the test range for any given temperature and accordingly consider the observed scatter. Methods are available to deal with such scatter, e.g. a percentage of outliers from the recommended scatter limits [27] or alternatively a "service condition-creep rupture property" interference (SCRI) model [28].

Comparisons of existing time–temperature parameters have been published frequently [11,16,29–31]. Interestingly, the basic parameters such as the LM parameter still give promising results despite their relative simplicity [11,30,31].

The main challenge of any creep model remains a reliable extrapolation of the rupture strength beyond the longest experimental time to rupture, especially for small data sets [11,16,29,32,33]. Reportedly, in some cases 50 experimental results are sufficient for reliable extrapolation but sometimes even 100 results do not give satisfactory predictions [27].

On the contrary, only a very limited data set of experimental results can be available. This is usually the case when new alloys are being tested [11] or if damage is predicted for thermomechanically loaded components where fatigue is expected to contribute more to the total damage than creep [9]. In these situations, the determination of master curves can only be based on an interim experimental data set (e.g. 9–20 experimental results, that is 3–5 stress levels at 3 or 4 test temperatures [34,35]). These tests are mostly carried out over a relatively short period of time, i.e. up to three months. As can be expected, the long-term predictions based on these results will probably not meet the requested criteria or can fall outside the critical boundaries. The aim of this paper is therefore to show how it is possible to accomplish a useful outcome where only a small set of experimental results is available.

The use of statistical tools on creep–rupture data is not an entirely new approach and has previously been attempted by many authors. Kim et al. [32] reported on the statistical properties of creep–rupture data. Holt and Bradford applied probabilistic modelling to predict the lifetime of nuclear boilers [36]. Remaining creep life predictions based on statistical models have been carried out by Baraldi et al. [37] whilst Evans set a statistical framework which enables the definition of confidence limits using the Wilshire equations [38]. Statistical processing has also been used for the evaluation of scatter in the experimental creep–rupture data [39] and extrapolation [16], however on a data set also including long-term results. Here, statistical processing is applied to short-term results only. While temperature and stress during a creep–rupture test must be supervised and maintained constant, their values cannot be varied in the search for the optimal values of coefficients. Hence the stress and the temperature are considered deterministic variables. However the time to rupture is observed with scatter as a rule and can thus be considered a random variable. Consequently, the predicted master curves determined from the experimental results can then also be considered random. A regularisation technique, i.e. smoothed bootstrapping [40,41] is then applied to the experimental data set. Using this technique and by employing input polynomials the number of input data can be virtually raised. By intensive re-sampling it is then possible to gain confidence intervals for the estimated creep–rupture strengths. Finally, medians are given as the estimates for creep–rupture strengths [16,36].

The RMB parameter has been used throughout this analysis as the main creep–rupture model. Additionally, the LM, the MH and OSD parameters have been considered and the predictions of the rupture strengths of the four parameters have been compared. However, any other creep model could be considered in the method instead.

Time–temperature parameters are compared on existing data sets of creep–rupture experimental results for ten metals [42]. The first and the second comparison in Section 3 serve as a reference for the results of the proposed method and have been conducted according to the European Creep Collaborative Committee (ECCC) recommendations [27]. In particular, the post assessment acceptability criteria for the creep–rupture data assessment have been considered by performing post assessment tests (PAT). Following these recommendations, three main evaluations have to be carried out:

- physical realism of the predicted master curves (PAT-1.1, PAT-1.2 and PAT-1.3), where master curves are checked visually,
- effectiveness of the model prediction within the range of the input data (PAT-2.1 and PAT-2.2), where diagrams of predicted creep–rupture times $\log t_r$ versus experimental creep–rupture times $\log t_{r,e}$ are examined, and
- repeatability and stability of the extrapolations (PAT-3.1 and PAT-3.2), where experimental results are randomly culled and the deviations of predicted rupture strengths are compared.

ECCC recommendations also provide some key recommendations for the assessment of sub-size creep–rupture data sets: at most 20 % deviance of master curves from the full-size data set and the rupture strengths at test temperatures should be reproduced to within 10 % for a given time to rupture. For further details on the ECCC recommendations, the reader is referred to [27].

2. Methodology

The method outlined in this section is summarised in Fig. 1.

A set of experimental results $\{T_i, \sigma_{ij,e}, t_{rij,e}\}^o$; $i = 1, \ldots, n_T$; $j = 1, \ldots, n_{\sigma i}$; $o = 1, \ldots, n_o$; where $t_{rij,e}$ and $\sigma_{ij,e}$ represent an experimental creep time to rupture and an experimental stress, respectively, is considered as an independent and identically distributed randomly chosen sample from all possible experimental outcomes.

n_T is the number of experimental temperatures, $n_{\sigma i}$ is the number of available results at i-th temperature and n_o is the number of chosen samples. For a test temperature T_i of the material of interest, an m-th order polynomial is fitted through logarithmic experimental results $\{T_i, \log t_{rij,e}, \log \sigma_{ij,e}\}^o$. A least-squares error function R^2_{1i} is defined as

$$R^2_{1i} = \sum_{j=1}^{n_{\sigma i}} \left(\log t^*_{rij,e} - \log t_{rij,e} \right)^2; \quad i = 1, \ldots, n_T, \tag{5}$$

where $\log t^*_{rij,e}$ represents the fitted logarithmic creep time to rupture. The latter is described by an m-th order polynomial as

$$\log t^*_{rij,e} = \sum_{k=0}^{m} b_{ik} (\log \sigma_{ij,e})^k. \tag{6}$$

Considering Eq. (6), the least-squares error function R^2_{1i} can be rewritten as

$$R^2_{1i} = \sum_{j=1}^{n_{\sigma i}} \left(\sum_{k=0}^{m} b_{ik} (\log \sigma_{ij,e})^k - \log t_{rij,e} \right)^2; \quad i = 1, \ldots, n_T. \tag{7}$$

Optimal values of coefficients b_{ik} can be found by minimising R^2_{1i},

$$\frac{\partial R^2_{1i}}{\partial b_{ik}} = \frac{\partial \sum_{j=1}^{n_{\sigma i}} \left(\sum_{k=0}^{m} b_{ik} (\log \sigma_{ij,e})^k - \log t_{rij,e} \right)^2}{\partial b_{ik}} = 0; \quad i = 1, \ldots, n_T. \tag{8}$$