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Three-dimensional free vibration analysis of functionally graded nanocomposite cylindrical panels reinforced by carbon nanotube

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ABSTRACT

This work deals with a study of the vibrational properties of functionally graded nanocomposite cylindrical panels reinforced by single-walled carbon nanotubes (SWCNTs) based on the three-dimensional theory of elasticity. The carbon nanotube reinforced (CNTRC) cylindrical panel have smooth variation of carbon nanotube (CNT) fraction in the radial direction and the material properties are estimated by the extended rule of mixture. Symmetric and asymmetric volume fraction profiles are provided in this paper for comparison. Suitable displacement functions that identically satisfy the boundary conditions at the simply supported edges are used to reduce the equilibrium equations to a set of coupled ordinary differential equation with variable coefficients, which can be solved by a generalized differential quadrature (GDQ) method. The results show that the kind of distribution and volume fraction of CNT have a significant effect on normalized natural frequency.

1. Introduction

In recent years, the development of nanocomposites has become an attractive new subject in materials science. Recent investigations have shown that carbon nanotubes possess remarkable mechanical properties, such as exceptionally high mechanical, thermal and electrical properties and even more important, a significant increase in fracture toughness [1–5], also demonstrated the orientation of the nanotubes plays an important role in the mechanical reinforcement [6–8]. The mechanical properties of carbon nanotube reinforced composites (CNTRCs) have been extensively investigated experimentally, analytically and numerically. Hu et al. [9] evaluated the macroscopic elastic properties of carbon nanotube-reinforced composites through analyzing the elastic deformation of a representative volume element under various loading conditions. Using molecular dynamic (MD) simulation, Han and Elliott [10] simulated the elastic properties of polymer/carbon nanotube composites. Wuite and Adali [11] found that the stiffness of CNTRC beams can be improved significantly by the homogeneous dispersion of a small percentage of CNTs. Vodenitcharova and Zhang [12] investigated the pure bending and bending-induced local buckling of CNTRC beams. However, the experimental and numerical studies concerning CNTRCs have shown that distributing CNTs uniformly as the reinforcements in the matrix can only achieve moderate improvement of the mechanical properties [13,14]. This is mainly due to the weak interface between the CNTs and the matrix where a significant material property mismatch exists.

Functionally graded materials (FGMs) are inhomogeneous composites characterized by smooth and continuous variations in both compositional profile and material properties and have found a wide range of applications in many industries. Currently some works have been done on continuously graded fiber reinforced through thickness. For example Sobhani Aragh and Yas [15] as well as Yas et al. [16] considered free vibrations of functionally graded fiber orientation and fiber reinforced cylindrical panels through using differential quadrature method. The results were presented for an orthotropic panel with arbitrary variations of fiber orientation and volume fraction through the thickness. Shen [17,18] suggested that the interfacial bonding strength can be improved through the use of a graded distribution of CNTs in the matrix and examined the nonlinear bending behavior of simply supported, functionally graded nanocomposite plates reinforced by single wall nanotubes (SWNTs) subjected to a transverse uniform or sinusoidal load in thermal environment. He also investigated postbuckling of nanocomposite cylindrical shells reinforced by SWCNTs subjected to axial compression in thermal environment and showed that the linear functionally graded reinforcements can increase the buckling load. Reviewing the literature on FG-CNTRC, it is realized most studies on CNT-reinforced composites (CNTRCs) have focused on their material properties.

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However some works are devoted to analysis of plates and shells made of FG-CNTRC [19–23]. Wang and Shen [24] investigated the large amplitude vibration and the nonlinear bending of a sandwich plate with CNT-reinforced composite face sheets resting on an elastic foundation on the basis of a micromechanical model and multi-scale approach. Nonlinear free vibration of functionally graded nanocomposite beams reinforced by aligned, straight single wall carbon nanotubes (SWCNTs) based on Timoshenko beam was investigated by Ke et al. [25].

In this paper, free vibration of functionally graded nanocomposite cylindrical panel reinforced by carbon nanotube is investigated using generalized differential quadrature (GDQ) method. Material properties are assumed to vary continuously along thickness direction. The effective material properties of functionally graded carbon nanotube are estimated using a micro-mechanical model. Micromechanics equations cannot capture the scale difference between the nano and micro levels [5,26]. In order to overcome this difficulty, the efficiency parameter is defined and estimated by matching the Young's moduli of CNTRCs obtained by the extended rule of mixture to those obtained by molecular dynamic (MD) simulation [10]. The effects of CNT distribution and volume fraction on the free vibration characteristic of the cylindrical panel are investigated.

2. Problem description

2.1. Functionally graded carbon nanotube-reinforced composite

An FGM cylindrical panel with its coordinate system (r, θ, z) is shown in Fig. 1 where r, θ, z are in the radial, circumferential and axial directions of the panel.

The mechanical properties of polymer nanotube composites are studied and discussed here. The effective mechanical properties of the CNTRC cylindrical panel are obtained based on a micromechanical model according to [17]:

$$E_{11} = \eta_1 V_{cn} E_{11}^{cn} + V_m E^m \tag{1}$$

$$\frac{\eta_2}{E_{ii}} = \frac{V_{cn}}{E_{ii}^{cn}} + \frac{V_m}{E^m} \quad (i = 2, 3) \tag{2}$$

$$\frac{\eta_3}{G_{ij}} = \frac{V_{cn}}{G_{ij}^{cn}} + \frac{V_m}{G^m} \quad (ij = 12, 13 \text{ and } 23) \tag{3}$$

$$\upsilon_{ij} = V_{cn}\upsilon^{cn} + V_m \upsilon^m \quad (ij = 12, 13 \text{ and } 23) \tag{4}$$

$$\rho = V_{cn}\rho^{cn} + V_m \rho^m \tag{5}$$

where E_{ii}^{cn}, G_{ij}^{cn} υ^{cn} and ρ^{cn} are elasticity modulus, shear modulus, Poisson's ratio and density, respectively, of the carbon nanotube and E^m, G^m $\left(G^m = \frac{E^m}{2(1+\upsilon^m)}\right)$, υ^m and ρ^m are corresponding properties for the matrix. $\eta_j (j = 1, 2, 3)$ is the CNT efficiency parameter and can be computed by matching the elastic modulus of CNTRCs observed from the molecular dynamic simulation results with those obtained from rule of mixture.

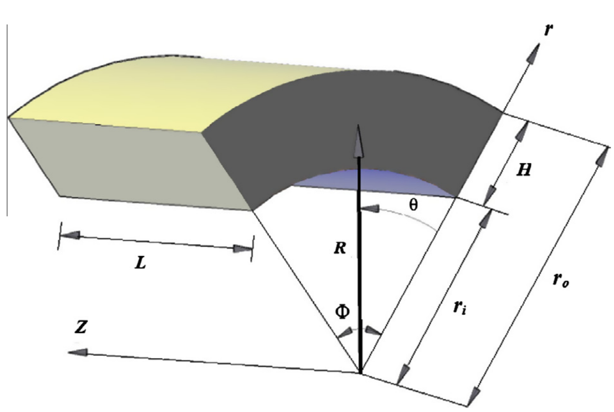

Fig. 1. Geometry of cylindrical panel.

The profile of the variation of the fiber volume fraction has important effects on the cylinder behavior. In this paper, the SWCNT reinforcement is either uniformly distributed (UD) or functionally graded in the thickness direction (FG), as shown in Fig. 2. It is assumed that the CNTRC cylindrical panel is made from a mixture of SWCNT and matrix which is assumed to be isotropic. For orthotropic cylindrical panel as shown in Fig. 2, the variation of the carbon nanotube volume fraction is assumed as follows:

$$\text{For type V}: V_{cn} = 2\left(\frac{r - r_i}{h}\right)V_{cn}^* \tag{6}$$

$$\text{For type } \Lambda: V_{cn} = 2\left(\frac{r_o - r}{h}\right)V_{cn}^* \tag{7}$$

$$\text{For type X}: V_{CN} = 4\left|\frac{r - R}{h}\right|V_{CN}^* \tag{8}$$

where

$$R = \left(\frac{r_i + r_o}{2}\right) \text{ and } V_{cn}^* = \frac{\rho^m}{\rho^m + \rho^{cn}(w^{cn})^{-1} - \rho^{cn}} \tag{9}$$

V_{cn} and V_m are the carbon nanotube and matrix volume fractions and are related by $V_{cn} + V_m = 1$.

2.2. The basic formulations

The mechanical constitutive relations, which relates the stresses to the strains are as follows:

$$\begin{bmatrix} \sigma_z \\ \sigma_\theta \\ \sigma_r \\ \tau_{r\theta} \\ \tau_{zr} \\ \tau_{z\theta} \end{bmatrix} = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & 0 & 0 & 0 \\ \bar{C}_{12} & \bar{C}_{22} & \bar{C}_{23} & 0 & 0 & 0 \\ \bar{C}_{13} & \bar{C}_{23} & \bar{C}_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{C}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{C}_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{C}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_z \\ \varepsilon_\theta \\ \varepsilon_r \\ \gamma_{r\theta} \\ \gamma_{zr} \\ \gamma_{z\theta} \end{bmatrix} \tag{10}$$

In the absence of body forces, the governing equations are as follows:

$$\begin{aligned} &\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{z\theta}}{r\partial \theta} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} = \rho\frac{\partial^2 u_z}{\partial t^2} \\ &\frac{\partial \tau_{\theta z}}{\partial z} + \frac{\partial \sigma_\theta}{r\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} = \rho\frac{\partial^2 u_\theta}{\partial t^2} \\ &\frac{\partial \tau_{zr}}{\partial z} + \frac{1}{r}\frac{\partial \tau_{\theta r}}{\partial \theta} + \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = \rho\frac{\partial^2 u_r}{\partial t^2} \end{aligned} \tag{11}$$

Strain–displacement relations are expressed as:

$$\begin{aligned} &\varepsilon_\theta = \frac{u_r}{r} + \frac{\partial u_\theta}{r\partial\theta}, \quad \varepsilon_r = \frac{\partial u_r}{\partial r}, \quad \varepsilon_z = \frac{\partial u_z}{\partial z}, \quad \gamma_{r\theta} \\ &= \frac{-u_\theta}{r} + \frac{\partial u_\theta}{\partial r} + \frac{\partial u_r}{r\partial\theta}, \quad \gamma_{zr} = \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \quad \gamma_{z\theta} = \frac{\partial u_\theta}{\partial z} + \frac{\partial u_z}{r\partial\theta} \end{aligned} \tag{12}$$

where u_r, u_θ and u_z are radial, circumferential and axial displacement components respectively.

Upon substitution Eq. (12) into (10) and then into (11), the following equations of motion as matrix form are obtained in term of displacement components:

$$\begin{bmatrix} K_{1r} & K_{1\theta} & K_{1z} \\ K_{2r} & K_{2\theta} & K_{2z} \\ K_{3r} & K_{3\theta} & K_{3z} \end{bmatrix} \begin{Bmatrix} u_r \\ u_\theta \\ u_z \end{Bmatrix} = \begin{Bmatrix} \rho\ddot{u}_r \\ \rho\ddot{u}_\theta \\ \rho\ddot{u}_z \end{Bmatrix} \tag{13}$$